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THE PERCENT. DEVIATION METHOD OF CONSTRUCTING VOLUME TABLES

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IN THE June number, the writer outlined certain refinements in plotting and harmonizing freehand curves, which may be applied to the conventional method of constructing volume tables. The proposals made, however, still leave one step in particular in that method lacking in the degree of precision that might be desired, viz., the determination of the slope of the regression lines of volume over height for the individual diameter classes. This is a phase of the work of constructing a volume table which is not dealt with very satisfactorily by any of the usual methods, in that either arbitrary assumptions are made regarding the relationship between volume and height, or the relationship is determined by more or less unsatisfactory graphic methods.

A new method of attack on the problem is embodied in the method to be outlined. The relation between volume and height for the different diameter classes is determined by a direct calculation. The only assumption made is that the regression is linear. In the majority of volume tables prepared by any method, the regression lines are either linear or if curved, the curvature is so slight as to make it doubtful whether the curvature is a general characteristic of the relationship.

In the proposed method, the graphic work is reduced to the plotting of two single curves, a third regression line which is required has been a linear one in the cases where the method has been tried and its equation was calculated. Offsetting the reduction in graphic work is an increase in arithmetical work, but the volume of this is considered to be less than in some of the newer volume table methods.

GENERAL VOLUME AND HEIGHT CURVES

The first step is the sorting of the data into diameter classes without regard to height. Separate curves of volume and height respectively over diameter are then plotted from the averages of the diameter classes, as in Fig. 1. These curves should be carefully balanced. The balancing of the curves is particularly important as there is no later correction for any errors

at this stage, and any errors in balancing will show up in the final check of the deviations of the tree volumes from the values of the final table.

RELATIONSHIP BETWEEN VOLUME AND HEIGHT

Volumes and heights are then read off the curves corresponding to the diameters of the individual trees taken to the nearest tenth of an inch and entered on a computation sheet, as shown in Table 1 for the fifteen inch diameter class.

TABLE 1
Computation of Moments for 15-inch Diameter Class.

D	H	H'e	Hc	dH	$\frac{x}{dH\%}$	$\frac{x^2}{(dH\%)^2}$	V	V'e	Vc	dV	$\frac{y}{dV\%}$	xy
14.6	81.1	79.3	79.4	+1.7	+2.1	4	37.2	37.0	36.8	+.4	+1.1	+2
14.8	81.1	79.8	79.9	+1.2	+1.5	2	41.9	38.1	37.9	+4.0	+10.6	+16
14.9	80.0	80.1	80.2	-.2	-.2	0	36.6	38.7	38.5	-1.9	-4.9	-1
14.9	79.5	80.1	80.2	-.7	-.9	1	40.4	38.7	38.5	+1.9	+4.9	+4
15.0	86.9	80.4	80.5	+6.4	+8.0	64	39.1	39.2	39.0	+.1	+.3	+2
15.0	80.9	80.4	80.5	+.4	+.5	0	39.8	39.2	39.0	+.8	+2.1	+1
15.0	83.7	80.4	80.5	+3.2	+4.0	16	40.3	39.2	39.0	+1.3	+3.3	+13
15.1	72.4	80.6	80.7	-8.3	-10.3	106	33.7	39.8	39.6	-5.9	-14.9	-153
15.1	76.3	80.6	80.7	-4.4	-5.5	30	37.0	39.8	39.6	-2.6	-6.6	-36
15.2	76.7	80.9	81.0	-4.3	-5.3	28	36.1	40.4	40.2	-4.1	-10.2	-54
15.2	80.5	80.9	81.0	-.5	-.6	0	42.8	40.4	40.2	+2.6	+6.5	+4
15.2	87.1	80.9	81.0	+6.1	+7.5	56	48.1	40.4	40.2	+7.9	+19.7	+148
15.3	82.1	81.2	81.3	+.8	+1.0	1	41.2	41.0	40.8	+.4	+1.0	+1
15.3	80.0	81.2	81.3	-1.3	-1.6	3	43.4	41.0	40.8	+2.6	+6.4	+10
15.5	85.1	81.7	81.8	+3.3	+4.0	16	41.8	42.2	42.0	-.2	-.5	-2
15.5	85.8	81.7	81.8	+4.0	+4.9	24	40.2	42.2	42.0	-1.8	-4.3	-21
1299.2		1291.8	+7.4	+9.1	351	639.6	634.1	+5.5	+13.7	+386		

$Hc = H'e + 0.1$, $Vc = V'e - 0.2$, $dH = H - Hc$, $dV = V - Vc$, $dH\% = dH/Hc \times 100$,
 $dV\% = dV/Vc \times 100$.

The curved volumes and heights are read from the preliminary curves as plotted before balancing, and are designated $H'e'$ and $V'e'$. The Hc and Vc values are secured by applying the corrections used in balancing the curves.

The deviations of the individual trees in height and in volume are next computed, and the deviations expressed as percentages of the curve values. The coefficient of linear regression of per cent. deviation in volume over per cent. deviation in height is then calculated. For this purpose the per cent. deviation in height may be designated as x and the per cent. deviation in volume as y , and the usual moments of linear regression, x^2 and xy , calculated. One decimal may be carried as shown, in the x and y values, but the resulting decimals in the squares and products may be dropped without sensible loss of accuracy. The decimals may readily be dropped in reading squares from a table of squares, and the product moments may be computed with a slide rule.

The following checks of the accuracy of the tabulation for each diameter class may be noted:

$$\begin{aligned}\Sigma dH &= \Sigma H - \Sigma H_c \\ \Sigma dV &= \Sigma V - \Sigma V_c \\ \Sigma dH\% &= \frac{\Sigma dH}{\overline{H}_c} \times 100 \text{ approximately.} \\ \Sigma dV\% &= \frac{\Sigma dV}{\overline{V}_c} \times 100 \text{ approximately.}\end{aligned}$$

To check x^2 and xy , it would be necessary to compute two additional columns, s and xs , where $s = x + y$. Then

$$\begin{aligned}\Sigma s &= \Sigma x + \Sigma y \\ \Sigma xs &= \Sigma x^2 + \Sigma xy.\end{aligned}$$

However, if the items in the x^2 and xy columns are checked approximately by mental arithmetic after the columns have been completed, gross errors may be detected and the complete check omitted.

The regression coefficients for the individual diameter class are next computed by the usual formula,

$$b = \frac{\Sigma xy}{\Sigma x^2}$$

For the fifteen inch diameter class,

$$\begin{aligned}b &= \frac{+386}{351} \\ &= +1.10\end{aligned}$$

This regression coefficient represents the average percentic difference in volume from the curve volume corresponding to a one per cent. difference in height from the curve height.

The values of b for all the diameter classes are given in Table 2 and plotted in Fig. 2. The data on which they are based are the same as were used for illustrative purposes in the previous article. It will be seen that there is a very considerable degree of variation, which indicates why it is so difficult in any volume table method, precisely to determine the correct relationship between volume and height.

TABLE 2
Computed Regression Coefficients

$\overline{D}$	n	b
5.2	13	0.09
6.1	21	0.48
7.2	36	0.98
8.1	34	1.43

9.0	38	1.08
10.1	45	1.08
11.0	31	0.56
12.1	32	1.14
13.0	28	1.48
14.1	19	0.47
15.1	16	1.10
15.8	9	1.84
17.0	7	0.87
18.2	3	0.66

333

It is next necessary to determine smoothed values of b for the different diameter classes and for this purpose a linear regression line is fitted by the method of least squares, weighting each value of b by the number of trees in its class. For the illustrative data, the regression equation is

$$b_c = 0.5887 + 0.0353D.$$

The values computed from this equation are given in the fifth column of Table 3.

While a curved regression line could be fitted if curvature were clearly indicated, the scatter of the computed values of b for the individual diameter classes has been so great in the cases investigated as to make it appear unlikely that this would often be justified. On theoretical grounds, it would seem that the conversion of the deviations in height and volume to percentages would tend to make the relationship an approximately linear one.

It may also be noted that the smoothed coefficients range from just below to just above one per cent. It is natural to expect that with total cubic volume, a one percent. difference in height should cause approximately one percent. difference in volume. With merchantable cubic volume, the coefficients should be somewhat larger, and considerably larger again in the case of board foot volume.

COMPUTATION OF FINAL VOLUME TABLE

This completes the determination of the values required for the computation of the final volume table. As a preliminary, the differences in volume corresponding to one foot differences in height are computed. Since the value of b_c represents for each integral diameter, the percent. difference in volume corresponding to a difference of one percent. of the curved height, then when it is multiplied by the percentage that one foot forms of the curved height, the volume percent. corresponding to an actual foot difference in height ($dV\%$)

is secured. This latter percent, when multiplied by the curved volume for the diameter (V_c) gives the actual difference in volume (dV) corresponding to one foot difference in height.

This value, multiplied by 10, is used in Table 4 to give the differences between the successive height classes in the final table. It is also used in the computation of the volume of any single height class as a further preliminary to the construction of the final table. This step is also illustrated in Table 3, where the volumes of the forty foot height class are computed. The difference between the curved height and forty feet is computed for each diameter and then multiplied by the difference in volume per foot. This product represents the amount by which the volume of a forty foot tree falls below (or above) the general curve volume, and is subtracted from (or added to) the curve volume to give the volume of a forty foot tree.

TABLE 3
Calculations Preliminary to Computation of Final Volume Table

D	H'e	Correc- tion	Hc	bc	dV% per foot	V'e	Correc- tion	Vc	dV per foot	Hc-40	dV (Hc-40)	Vc 40' class
5	40.3		40.3	.765	1.898	2.5		2.5	.047	0.3	.01	2.49
6	45.9		45.9	.800	1.743	4.0		4.0	.070	5.9	.41	3.59
7	51.2		51.2	.856	1.633	6.1	-.1	6.0	.098	11.2	1.10	4.90
8	56.2	+.1	56.3	.871	1.547	8.6	-.1	8.5	.131	16.3	2.14	6.36
9	60.7	+.2	60.9	.906	1.488	11.5	-.1	11.4	.170	20.9	3.55	7.85
10	64.8	+.3	65.1	.942	1.447	14.9	-.1	14.8	.214	25.1	5.37	9.43
11	68.4	+.2	68.6	.977	1.424	18.9	-.2	18.7	.266	28.6	7.61	11.09
12	71.7	+.1	71.8	1.012	1.409	23.3	-.2	23.1	.325	31.8	10.33	12.77
13	74.8		74.8	1.048	1.401	28.3	-.3	28.0	.392	34.8	13.64	14.36
14	77.7		77.7	1.083	1.394	33.5	-.2	33.3	.464	37.7	17.49	15.81
15	80.5		80.5	1.118	1.389	39.2	-.2	39.0	.542	40.5	21.95	17.05
16	83.2		83.2	1.154	1.387	45.2	-.2	45.0	.624	43.2	26.96	18.04
17	85.8		85.8	1.189	1.386	51.4	-.1	51.3	.711	45.8	32.56	18.74
18	88.3		88.3	1.224	1.386	57.8		57.8	.801	48.3	38.69	19.11
19	90.7		90.7	1.259	1.388	64.5		64.5	.895	50.7	45.38	19.12

$$dV\% \text{ (per foot)} = bc \times \frac{1}{Hc} \times 100$$

$$dV \text{ (per foot)} = dV\% \text{ (per foot)} \times \frac{Vc}{100}$$

$$dV \text{ (Hc-40')} = dV \text{ (per foot)} \times (Hc-40')$$

$$Vc \text{ (40' class)} = Vc - dV \text{ (Hc-40')}$$

The final table is then completed by adding to (or subtracting from) the volume for a forty foot tree, the difference in volume for each ten foot difference in height as shown in Table 4. An extra decimal should be carried at this stage and rounded off in the final table, when the volumes computed for unnecessary short height-classes in the larger diameter classes may be dropped.

TABLE 4
Computation of Final Volume Table

D	av per 10'	Total height in feet						
		30	40	50	60	70	80	90
		Total volume in cubic feet						
5	.47	2.02	2.49	2.96	3.43			
6	.70	2.89	3.59	4.29	4.99			
7	.98	3.92	4.90	5.88	6.86			
8	1.31		6.36	7.67	8.98	10.29		
9	1.70		7.85	9.55	11.25	12.95		
10	2.14		9.43	11.57	13.71	15.85	17.99	
11	2.66		11.09	13.75	16.41	19.07	21.73	24.39
12	3.25		12.77	16.02	19.27	22.52	25.77	29.02
13	3.92		14.36	18.28	22.20	26.12	30.04	33.96
14	4.64		15.81	20.45	25.09	29.73	34.37	39.01

Diameter classes above 14" omitted.

Heavy lines enclose figures included in final table.

CHECK OF FINAL TABLE

Volumes corresponding to the mean diameters and heights of the trees in each size-class, which were computed for this purpose, were secured by double interpolation in the final table. The deviation of the mean volume of each size-class from the corresponding interpolated tabular volume was multiplied by the number of trees in the class and the results are summarized in Table 5.

TABLE 5
Net Deviation of Tree Volumes from Tabular Volumes

Diameter Group	Height Class							Total
	30	40	50	60	70	80	90	
Aggregate Net Deviation								
5- 7"	—0.1	+1.0	—6.5					—5.6
8-10"		—0.6	—9.6	+0.5	+3.3			—6.4
11-13"				—12.3	+7.5	+4.2		—0.6
14-18"					—4.6	+5.6	—6.1	—5.1
Total	—0.1	+0.4	—16.1	—11.8	+6.2	+9.8	—6.1	—17.7
After Correction								
5- 7"	+0.2	+3.7	—3.8					+0.1
8-10"		—0.1	—6.8	+4.3	+3.4			+0.8
11-13"				—12.3	+7.5	+4.2		—0.6
14-18"					—3.9	+8.2	—4.0	+0.3
Total	+0.2	+3.6	—10.6	—8.0	+7.0	+12.4	—4.0	+0.6

The aggregate net deviation is—17.7 cu. ft. or 0.29% of the total volume of 6091 cu. ft. Reduction in the volume of the 6, 7, 8, 9, 15 and 16 inch trees by 0.1 cu. ft., the 17 inch trees by 0.2 cu. ft. and the 18 inch trees by 0.5 cu. ft. reduces the net deviation to 0.6 cu. ft. and leaves the net deviations of the diameter groups almost perfectly balanced.

The net deviations of the individual height classes in the separate diameter groups indicate how well the regression of volume over height is taken care of. A plus sign on the left and a minus sign on the right indicates that the regression lines of volume over height would if plotted be too steep, and vice versa, too flat. The indications in Table 5 are:

5- 7"—too steep	11-13"—too flat
8-10"—too flat	14-18"—about right.

These discrepancies correspond to the differences between the average actual regression coefficients of volume on height and the smoothed regression coefficient for the average diameters of these groups. The comparison is as follows:

Diameter Group	Actual Average Regression Coefficient	Smoothed Regression Coefficient
5- 7"	0.67	0.82
8-10"	1.18	0.91
11-13"	1.05	1.01
14-18"	0.95	1.13

Where the actual average regression coefficient is lower than the smoothed coefficient, the basic data will naturally show less difference in volume with difference in height and will be higher than the tabular volumes for the shorter trees and lower for the taller. In other words there will be a net positive deviation for the shorter trees and a net negative deviation for the taller. This is true for the first group and the opposite is true of the two middle groups. The situation could be improved only by curving the regression line of Fig. 2 which does not seem to be justified in view of the scatter of the individual points.

CONCLUSION

The method as outlined involves considerably more arithmetical work than the method of harmonized curves, but on the other hand, the lack of precision involved in plotting and harmonizing partial regression lines of volume over diameter for individual height classes is avoided. This would be a very great advantage were a smaller number of trees involved, in which case the partial regression lines might be poorly defined. In such a case, the total

regression lines of volume and height over diameter, which are all that are required in this method, would still be reasonably well defined, and with proper balancing could be drawn with a fair degree of precision. The succeeding work of determining the regression of volume over height is purely arithmetical and is carried out in such a way as to render unnecessary the use of personal judgment to any extent at all.

A further advantage of the method is that no variation in technique is necessary when working with small or large numbers of trees, with different volume units such as cubic feet or board feet, or when different types of volume tables are to be constructed. The construction of merchantable length volume tables requires merely the substitution of merchantable length for total height in the computations outlined above and the plotting of a curve of merchantable length over diameter in place of the curve of total height over diameter as in Fig. 1.

The method was adapted from Bruce and Reinecke's* method of constructing basal area tables and total cubic foot yield tables. In those cases however, there is no correlation between the two independent variables—site-index and age. In the case of a volume table on the other hand, height and diameter are strongly correlated, which requires some modification of the method. The computation of different regression coefficients of volume over height for the different diameter classes is similar in its effect to Bruce & Schumacher's* modification of Bruce's earlier method.

SUMMARY

A method of constructing volume tables has been described which involves the plotting of only general curves of height and volume over diameter alone, and the computation of linear regression coefficients showing for each diameter class the relationship between the percentic deviations in volume and the percentic deviations in height from these general curves. The regression coefficients are then correlated with diameter and smoothed values computed. From these smoothed regression coefficients and the curved heights and volumes of the different diameter classes, the final volume table is computed.

The method involves more arithmetical work but much less graphic plotting and personal judgment than the harmonized curves method and is better adapted to the construction of a volume table from the measurements of a small number of trees.

*Donald Bruce—A method of preparing timber-yield tables. *Jrl. Agric. Res.* Vol. 32, No. 6.

*L. H. Reinecke—A modification of Bruce's method of preparing timber-yield tables. *Jrl. Agric. Res.* Vol. 35, No. 9.

*Donald Bruce and Francis X. Schumacher—*Forest Mensuration*. McGraw-Hill Book Co., N.Y., pp. 211-218.

*James G. Osborne and Francis X. Schumacher—The construction of normal-yield and stand tables for even-aged timber stands. *Jrl. Agric. Res.* Vol. 51, No. 6.