EIGENVECTOR ANALYSES SHOW THAT BIRCH AND PINE HAVE SIMILAR
FORM IN SWEDEN AND BRITISH COLUMBIABY JORAN FRIES¹

The aim of the study was to find the pattern of variation in stem form for different species. Professor Matern suggested the calculation of eigenvalues and eigenvectors for this purpose. A general description of this method may be found in textbooks on linear algebra (e.g. Murdoch, 1959); therefore this report deals only with the application of this method to the present study. Copies of the IBM7040 computer programs used can be obtained from the U.B.C. Faculty of Forestry.

Since diameter inside bark (dib) measured at the same positions in each tree forms a matrix of dib on height, a series of eigenvectors calculated from the matrix can be used to define and compare the form of various classes of trees. The characteristic roots of a square matrix are called latent roots or eigenvalues. Eigenvectors are non-zero vectors of any n-th order square matrix.

The data consisted of measurements of diameters for felled trees. The points of measurement were situated at certain percentages of the total height of the tree. At each point of measurement the diameter outside bark and the double bark thickness were recorded. All calculations were made for diameters inside bark.

Eigenvalues and eigenvectors were calculated for 536 birch *Betula verrucosa* Ehrh. and 2,700 Scots pine *Pinus silvestris* L. from Sweden and for 75 white birch *Betula papyrifera* var. *commutata* (Regel) Fern. as well as 450 lodgepole pine *Pinus contorta* var. *latifolia* Engelm. from the Interior of British Columbia near Prince George.

The diameters used in the calculations were obtained at 4, 10, 20, 30, 40, 50, 60, 70, 80, and 90 per cent of the total tree height, or were interpolated to these positions.

Figures 1a-c and 2a-c show the first three eigenvectors for pine and birch, respectively. These are the three eigenvectors with the highest eigenvalues. Values for each position in the tree are connected with straight lines to make the figures easier to understand.

The first eigenvector gives the linear relationship between dib's. This is the mean stem form for all trees in the group. The second eigenvector gives the linear relationship between the deviations from the first eigenvector. The third eigenvector gives the linear relationship between the deviations from the second eigenvector and so on. The eigenvalues for the third eigenvectors are only about $\frac{1}{3}$ to $\frac{1}{4}$ of the eigenvalues for the second eigenvectors. This means that for describing stem form the third eigenvector is considerably less important than the second eigenvector. Higher order eigenvectors have successively lower eigenvalues and are consequently less important than eigenvector number three.

¹ Dr. Fries is Docent, Royal School of Forestry, Stockholm, Sweden. This report summarizes his work with Dr. J. H. G. Smith as a Research Associate, U.B.C. Faculty of Forestry, Vancouver 8, B.C. Financial support of the National Research Council of Canada is gratefully acknowledged, and provision of data by the B.C. Forest Service, Surveys and Inventory Division is much appreciated.

The figures show that there is a great similarity between the eigenvectors for pine from B.C. and pine from Sweden. The same is also true for the eigenvectors for birch from B.C. and from Sweden.

The pine and birch trees from Sweden were sorted into five groups according to their crown length per cent, the length of the living crown divided by the total height of the tree. For each species and each group, eigenvalues and eigenvectors were calculated. Between the different groups there was a significant difference in the values of eigenvector No. 1. Those trees having shorter crowns showed less taper than those having longer crowns (Fig. 3a). This association of reduced taper with decrease in live crown length has been established for most species. The differences between the corresponding element of the eigenvectors No. 1 for trees with short and long crowns showed, however, a strong correlation with the corresponding elements of eigenvector No. 2. This means that diameters of trees with varying taper associated with varying crown lengths can be described in terms of the appropriate eigenvectors and heights. The constants may be regarded as a size factor highly correlated with the dbh of the tree and a form factor correlated with percentage live crown.

A sorting of the trees according to dbh/height ratios showed a similar pattern for the elements of eigenvector No. 1 (Fig. 3b). This means that the form factor is also correlated with the dbh/height ratio. If the elements of eigenvectors 1 and 2 are fitted into functions the diameter can be calculated for any height in the tree.

Eigenvalues and eigenvectors calculated for the stem form of trees can be used for several purposes. One application will be to study the stem form and its connection with the strength properties of the wood and the stress applied to the stem by wind pressure and weight. The eigenvectors will also be used for the calculation of diameters at given heights in trees of various size and form and to calculate the volume up to these given heights. A more extensive report describing the theory and applications of this technique may be published by the Royal College of Forestry, Stockholm, Sweden. Analyses will be extended to other commercial species of British Columbia.

REFERENCE

MURDOCH, D. C. 1959. Linear algebra for undergraduates. Wiley, 239p.