Interpolation Errors on Some Stem Profiles

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Introduction

Stem profiles are used in studies of tree volume, form and taper; they are generally constructed by plotting stem diameters measured at various heights of the tree over these heights, and by drawing a smooth curve through the resulting points. Stem diameters are sometimes required at points other than those measured. Since additional field work is costly, the investigator often resorts to interpolation of existing data. The interpolated diameters can be expected to differ from the actual ones, and to introduce a source of error into the investigations. The purpose of this study is to evaluate the source, magnitude and significance of these interpolation errors.

Most sources of interpolation errors are fairly evident; they include method of interpolation, position of interpolated points on the stem profile and length of interpolation (the distance between the measured point and the interpolation point).

Two methods of interpolation were tested: in one, a polynomial was fitted to go through a series of upper stem diameter points; in the other, interpolation was on a straight line between two points.

Procedure

Tree section measurements of 20 red pine trees (*Pinus resinosa* Ait.) were used in the study. They were selected at random from approximately 1000 sets of tree data. The data used for each tree were:

- total tree height (H);
- stem diameter at breast height (D), outside bark;
- outside bark diameters (d) measured at stump height and at nine points evenly spaced between breast height and the tip of the tree.

The trees ranged between 3 and 22 inches in stem diameter at breast height and from 22 to 102 feet in height.

Fig. 1 shows the measured points on a stem profile. For each tree, measurement points 3, 5, 7, 9 and 11 were used to fit a polynomial and four straight lines; measurement points 4, 6, 8 and 10 provided a basis for comparison with the interpolated values. For the 20 trees, this procedure yielded 80 interpolated diameters for each of the two interpolation methods, as well as the 80 actual diameters used for comparison. The differences between actual and interpolated diameters (i.e. interpolation errors) were used in subsequent analyses.

Figure 1. Stem profile of tree no. 181.

Analyses and Results

The interpolation errors were compiled separately for each interpolation method. Statistics are presented in Table 1.

Table 1 shows little difference between the two methods for "Range" and "Standard deviation". However, the "Mean" for the straight-line method is considerably larger than that for the polynomial method. The results of the t-test further show that the interpolation errors differ significantly from zero for the straight-line method, but not for the polynomial method.

Table 1

Diameter Interpolation Errors

Statistics	Polynomial method	Straight-line method
	(inches)	(inches)
Range	-1.28 to +1.16	-1.22 to +1.02
Standard deviation	±.40	±.35
Mean	-.017	+.095
SD of mean	±.045	±.039
"t" statistic	.378 n.s.	2.44*

n.s. : not significant

* : significant at the 5% level

Results from a separate analysis of variance also showed that the interpolation errors of the two methods differ significantly. In the same analysis, the effect of position of interpolated points on the stem profile on interpolation error was tested; highly significant differences were found between errors in the four positions. Although further statistical analyses did not reveal any trend, a visual inspection of the stem profiles showed that the straight-line method underestimated actual dia-

meters at positions 3 and 4 (closest to the tip of the tree); this underestimate seems logical, since the curvature of a stem profile is greater at these points. The visual inspection also revealed that the stem profiles were quite irregular; this caused large random errors.

The effect of interpolation length (distance between interpolation point and measured point) on interpolation error was examined in a series of regression analyses. The absolute values (sign ignored) of the interpolation errors E were related to interpolation lengths L . A single linear regression was sufficient for both interpolation methods. The resulting equation had the form

$$E = -.0164 + .0245L$$

The correlation coefficient $r = .462$ and the regression coefficient were both significant at the 1% level.

Summary and Conclusion

Interpolation errors can be attributed to:

- irregular stem profile (probably due to knots or nodal swelling) causing fairly large random errors;
- a small but significant bias of the straight-line method, most noticeable at interpolation points on the upper part of the stem;
- interpolation length.

Although several sources have been found to cause significant interpolation errors, the researcher himself must decide whether or not the errors are acceptable. This study will provide him with some information on which to base his judgment.