Stocking of Northern Hardwood Regeneration Based on Exponential Dropout Rate

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Abstract

Survival and mortality in even-aged seedling-sapling northern hardwoods are described as an exponential process, where the probability of a tree — alive at mean stand diameter $\bar{D}_0$ and living to mean stand diameter $\bar{D}_1$ or beyond — is:

$$P = e^{-R(\bar{D}_0 - \bar{D}_1)}$$

Values of R were estimated from field data. The average number of trees at $\bar{D}_0$ required to produce 1 tree at $\bar{D}_1$ equals $1/P$. By using the binomial distribution, the author determined the number of trees at $\bar{D}_0$ required to produce with high probability 1 tree or more at $\bar{D}_1$. Regeneration stocking guides, based on this mathematical process, are presented.

Résumé

Normes de stocking dans les forêts feuillues du Nord américain où l'élimination des tiges suit le modèle d'une fonction exponentielle

La survie et la mortalité des tiges dans les jeunes peuplements feuillus du Nord, au stade du semis-gaulis peuvent être décrites au moyen d'un modèle mathématique à fonction exponentielle dans lequel la probabilité qu'un brin, vivant à l'époque décrite par $\bar{D}_0$ survive et se prolonge par delà l'époque décrite par $\bar{D}_1$, est:

$$P = e^{-R(\bar{D}_0 - \bar{D}_1)}$$

Les valeurs de R ont été estimées d'après les observations prises sur le terrain. Le nombre moyen de brins requis à l'époque $\bar{D}_0$ pour produire une tige en $\bar{D}_1$, est égal à $1/P$. À l'aide de la

distribution normale, l'auteur a pu déterminer le nombre de brins requis à l'époque $\bar{D}_0$ pour produire avec une probabilité élevée, une tige ou davantage à l'époque $\bar{D}_1$. Grâce à ce modèle, l'auteur nous offre des normes de stocking comme jalons pratiques pour ces études de régénération.

Stocking and density are important concepts in both the theory and practice of silviculture. And an understanding of the one concept is essential to an understanding of the other.

According to Bickford (1957), stand density is a numerical expression of those physical characteristics of a stand that denote crowding or utilization of the site. Stocking, on the other hand, is a numerical expression of actual density — as compared with some ideal, desired, or normal density — based on growth, development, or management considerations.

Density in seedling-sapling stands often is expressed in terms of numbers of stems. Density is also evaluated by "stocked plot" (stocked quadrat) where the size of plot and definition of "stocked" may vary to suit the need. Although the term "stocked-plot" implies a measurement of stocking, the relationship of numbers or percentages of "stocked-plots" to some norm or objective frequently is not clear. Thus, the "stocked-plot" method commonly provides a measure of density rather than of stocking.

New measurement techniques, such as "distance" methods (Persson, 1964), are being used increasingly to measure density in young stands.

The evaluation of stocking is more difficult. No method of evaluating stocking has been accepted for seedling-sapling northern hardwoods. The required biological information is scarce. But even

more important, a logical theory for evaluating stocking in these young stands is needed.

However, with the advent of better stocking guides for poletimber and larger stands (e.g., Roach and Gingrich, 1962; Gingrich, 1967) coupled with the application of statistical theory, new insight has been gained into the problem of defining and measuring stocking in seedling-sapling stands. This paper describes both the preliminary theory and the development of specific stocking guides for northern hardwood species.

Theory

As a young stand of northern hardwoods increases in age, mean diameter increases and number of trees decreases because of mortality. Number of trees in a stand plotted over mean stand diameter forms a typical inverted J-shaped curve. Suppose we had a series of stands that averaged 0.0, 0.1, 0.2... 5.0, 5.1... 20.0, 20.1... etc. inches in mean diameter. If we drew individual trees at random from this series of stands, the probability of drawing a tree in a stand of mean diameter D would be approximated by the negative exponential probability distribution. Thus, we are justified in saying that mean diameter (measured on a continuous scale) in an artificially constructed series of stands follows the negative exponential probability distribution:

$$P = e^{-R(\bar{D}_0 - \bar{D}_1)}$$

- (1) $f(\bar{D}_1) = Re^{-R(\bar{D}_0 - \bar{D}_1)}$
 where $\bar{D}_0$ is the minimum (threshold) mean diameter considered
 $\bar{D}_1$ is mean stand diameter
 $e = 2.71828$, the base for natural logarithms.
 R = the parameter of the distribution, which reflects tree mortality rate.

The cumulative probability distribution is:

$$(2) F(\bar{D}_1) = 1 - e^{-R(\bar{D}_0 - \bar{D}_1)}$$

Equation (2) defines the probability that, beginning with a minimum mean diameter D_0 , a single selected value of mean diameter will fall between D_0 and D_1 . Equation (2) also represents the chance that a single stem present at D_0 will die before the stand reaches D_1 . It follows that the probability of a tree, present at D_0 and living to or beyond D_1 , is:

$$(3) P = e^{-R(\bar{D}_0 - \bar{D}_1)}$$

Assuming that trees die independently of one another with equal probabilities, the binomial distribution can be used to determine the probability that n_0 trees at mean diameter D_0 will produce n_1 trees at mean diameter D_1 .

$$(4) P(n_1) = \frac{n_0!}{n_1!(n_0 - n_1)!} \cdot P^{n_1} \cdot Q^{(n_0 - n_1)}$$

Where $Q = 1 - P$ (equation 3)

Equations (3) and (4) represent a simple "death process" (e.g., Bailey, 1964) where the chances of mortality are related to mean diameter instead of the more usual variable of real time.

Next, consider how equation (4) can be used in developing seedling-sapling stocking guides. Available stocking guides for poletimber and larger stands invariably indicate — among other things — the minimum numbers of stems and basal areas for given mean diameters. For example, suppose that we desired a minimum of 400 well-distributed acceptable stems (55 square feet basal area) per acre when the stand reached 5.0 inches d.b.h. This means that we would like to achieve 1 tree for each 108.9 square feet of stand area ($43,560 \div 400$). Thus, the appropriate sampling unit is a plot that contains 108.9 square feet. Then equation (4) can be used to determine the probability that n_0 initial trees will produce $n_1 \geq 1$ tree. In practice, this computation is best performed by setting $n_1 = 0$, and then using $1 - P(0)$ as the probability of encountering 1 or more trees.

In the field, a plot would be classed as stocked if n_0 were sufficiently large to produce $n_1 \geq 1$ tree with high probability. The ideal would be to have all plots classed as stocked. Thus, the percentage of plots stocked would indeed be a measure of stocking.

Field data

To apply the theory just described, it is necessary to obtain an estimate of the parameter R used in equations (1) — (3). To do this, field data were taken as follows:

Eighteen plots were located in typical young even-aged northern hardwoods on the White Mountain National Forest, New Hampshire. Average stand diameter ranged from about 1.0 to 4.5 inches d.b.h. (trees 0.6 inch and larger). The stands ranged in age from about 10 to 40 years. Five plots — those in the youngest stands — were 1/20 acre in size; the others were 1/10 acre. On each plot, all trees 0.6 inch d.b.h. and larger were tallied by species and d.b.h. to the nearest 0.1 inch.

Species composition, based on number of stems, ran fairly heavily to tolerant species: 53 percent sugar maple and beech, 7 percent yellow birch, 9 percent red maple, 9 percent white ash, 8 percent paper birch, and 14 percent miscellaneous species.

Computations

First, we worked with the so-called commercial hardwoods: beech, sugar maple, yellow birch, red maple, white ash, and paper birch.

A regression of $Y = a + bX$ was fitted, where:

Y = logarithm (base 10) of number of trees 0.6 inch +

X = mean stand diameter

The resulting equation was:

$$\text{Log}_{10} N = 3.695129 - 0.183231 \bar{D}$$

The correlation coefficient was 0.92.

Based on the known relationships between R , q (De Liocourt's q), and the regression coefficient b (Leak, 1963 and 1965), the parameter R was estimated from the relationship:

$$\begin{aligned} q &= \text{antilog}_{10} b \\ q &= e^R \\ \therefore e^R &= \text{antilog}_{10} b. \end{aligned}$$

By this means, R was estimated as 0.42 for all commercial species combined.

Sometimes we are interested in evaluating regeneration stocking in stands dominated by less tolerant species. Thus the same estimation procedure was applied to the numbers of trees and mean diameters for the intolerant and intermediate species combined: yellow birch, red maple, white ash, and paper birch.

The regression equation, with a correlation coefficient of 0.91, was:

$$\text{Log}_{10} N = 3.34378 - 0.14408 \bar{D}$$

The parameter R for intolerant and intermediate species was estimated at 0.33. This value was considerably lower than the value for all species combined. This indicated that mortality was less for the intolerants and intermediates for a given change in mean diameter; it did not indicate that mortality over time was less.

Results

Using equation (3), the probabilities of a tree surviving over a given change in mean diameter were calculated for $R=0.42$ (all commercial species) and $R=0.33$ (intolerants and intermediates) (Table 1).

Table 1 Probability of a tree being alive at a given future mean stand d.b.h., assuming that the tree was present at a given initial stand d.b.h.

Initial mean d.b.h.	Future mean d.b.h.				
	1.0	2.0	3.0	4.0	5.0
All commercial species ($R = 0.42$)					
1.0	1.0	0.66	0.43	0.28	0.19
2.0	—	—	.66	.43	.28
3.0	—	—	—	.66	.43
4.0	—	—	—	—	.66
Intolerants and intermediates ($R = 0.33$)					
1.0	1.0	0.72	0.52	0.37	0.27
2.0	—	—	.72	.52	.37
3.0	—	—	—	.72	.52
4.0	—	—	—	—	.72

The average number of trees required to produce one tree at some future mean diameter was approximated by taking the reciprocal of the probabilities in Table 1. These average numbers are given in Table 2.

Table 2 Average number of trees, at a given initial mean stand diameter, required to produce 1 tree at a given future mean diameter

Initial mean d.b.h.	Future mean d.b.h.				
	1.0	2.0	3.0	4.0	5.0
All commercial species					
1.0	1.0	1.5	2.3	3.6	5.3
2.0	—	—	1.5	2.3	3.6
3.0	—	—	—	1.5	2.3
4.0	—	—	—	—	1.5
Intolerants and intermediates					
1.0	1.0	1.4	1.9	2.7	3.7
2.0	—	—	1.4	1.9	2.7
3.0	—	—	—	1.4	1.9
4.0	—	—	—	—	1.4

The number of trees required to produce, with a probability of 0.85 or greater, one or more trees at some future mean diameter was determined. We used the probabilities in Table 1 and based our calculations on the binomial distribution (equation 4). These high-confidence numbers are given in Table 3.

The results in Tables 2 and 3 can be used to judge the adequacy of stocking in seedling-sapling stands. However, we must first estimate the desired minimum number of stems per acre at some future diameter. For example, assume that 400 stems at a mean stand diameter of 5.0 inches was the goal.

If our information on the stand in question consisted of only an estimate of current mean diameter and numbers of trees per acre, we could use Table 2 to judge the adequacy of stocking. For example, if we were dealing with a stand averaging about 1.0 inch in diameter and composed of a mixture of species, Table 2 indicates that an average of about $5.3 \times 400 = 2,120$ acceptable stems per acre would be needed to produce adequate stocking. The comparable figure for a stand (or portion of a stand) composed of intolerants and intermediates would be $3.7 \times 400 = 1,480$.

On a more intensive basis, we could run a stocking survey as follows. The goal of 400 stems per acre is equivalent to 1 stem per 108.9 square feet. Thus, the appropriate sampling unit would be a

Table 3 Number of trees, at a given initial mean diameter, required to produce 1 tree or more at a given future mean diameter with a probability of approximately 0.85 or greater

Initial mean d.b.h.	Future mean d.b.h.				
	1.0	2.0	3.0	4.0	5.0
All commercial species					
1.0	1	2	4	6	9
2.0	—	—	2	4	6
3.0	—	—	—	2	4
4.0	—	—	—	—	2
Intolerants and intermediates					
1.0	1	2	3	5	6
2.0	—	—	2	3	5
3.0	—	—	—	2	3
4.0	—	—	—	—	2

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square plot 10.4-foot on each side, or a circular plot with a radius of 6.1 feet. For a 1.0-inch stand, the plot would be judged as stocked **on the average** if it contained 5 (5.3) (all species) or 4 (3.7) (intolerants and intermediates) stems (Table 2). However, to be judged as stocked with a high degree of confidence, we would require that the plot contain 9 (all species) or 6 (intolerants and intermediates) acceptable stems (Table 3). We recommend using the high-confidence figures. The percent of plots classed as stocked would be a measure of percentage of stocking.

Discussion

Survival and mortality in young forest stands is a dynamic, biological process. However, it appears as though the process can be accurately simulated by the mathematical model described above.

Use of the binomial distribution, involves the assumption that trees die independently of one another with equal probability. The assumption of independence in any biological community is open

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to question. However, the use of this assumption with other statistical methodology involving individual trees has not, to my knowledge, let to serious misinterpretation.

Trees of different species, vigors, etc. differ in survival-mortality probabilities. Thus, the use of equal probabilities is a simplification considered necessary at this stage in the development of the model. Given the necessary biological information, we could no doubt use varying probabilities to define more exactly the survival-mortality process and thus to develop more refined stocking guides.

The results in this paper apply to undisturbed natural stands of northern hardwoods. The same approach could be applied to stands that have been weeded or cleaned. However, survival probabilities would no doubt be higher than those reported above, in proportion to the intensity of the weeding operation. In fact, if selected crop trees were carefully freed from competition, it is conceivable that survival probabilities would approach the value of 1.0.

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