Taper equations can be converted to volume equations and point sampling factors

J. P. DEMAERSCHALK¹

Faculty of Forestry
University of British Columbia
Vancouver 8, B.C.

Certain tree taper equations can be converted easily to volume equations and point sampling factors. Such equations have the advantage that identical solutions for tree volume are obtained, independent of the type of equation used.

The accuracy of the resulting volume equation depends on the degree of bias of the taper equation. The effect of bias varies considerably with its position on the stem and with the size of the tree (Demaerschalk 1971). Estimation of section diameters of butt logs and of large trees is most difficult. It is not possible to specify the actual accuracy of these taper based equations because the original data have not been available for analyses of volume based on the same samples. It is likely that estimates of volume based on taper will agree reasonably well with direct determinations of volume but further study is needed.

Kozak et al. (1969a, b) proposed, developed and tested a taper equation, based on the parabolic function:

$$\frac{d^2}{D^2} = a + b \frac{h}{H} + c \frac{h^2}{H^2}$$

where h is the height above the ground in feet, d is the diameter inside bark in inches at any given h in feet, H is the total height of the tree in feet, D is the diameter breast height outside bark, in inches, and a , b and c are the regression coefficients.

After integrating this equation to volume the following is obtained:

$$V = K_1 \cdot D^3 \cdot H$$

where V is the total volume of the tree, inside bark, in cubic feet and K_1 is a constant. This is the familiar combined variable volume equation with intercept equal to zero. The coefficient K_1 is calculated, using the formula:

$$K_1 = 0.00545415 (a + b/2 + c/3)$$

where the coefficients a , b and c are the regression coefficients from the taper equation.

From this combined variable volume equation, ratios of volume to basal area can be derived easily from:

$$V/B = K_2 \cdot H$$

where B is basal area of the tree at breast height outside bark, in square feet, and the constant K_2

is calculated as:

$$K_2 = (a + b/2 + c/3)$$

Because of the zero intercept in the volume equation, the point sampling factor derived from it is independent of D . This makes the formula simple to calculate.

In this note, the values of K_1 and K_2 , for British Columbia tree species (Table 1), are derived from the taper equation coefficients of Kozak et al. (1969b). The values of K_2 in Table 1 are similar to those given by Smith and Munro (1965) who derived point sampling factors from combined variable volume equations, neglecting the intercept. Differences are, of course, to be expected because the point sampling factor coefficients in Table 1 are derived from taper equations based on taper lines of the British Columbia Forest Service (Kozak et al. 1969b), while the point sampling factors, given by Smith and Munro (1965), are indirectly derived from the British Columbia Forest Service logarithmic tree volume equations (Browne 1962). Browne's equations represented all age classes for 16 of the species groups which have been represented by either mature or immature taper lines in Table 1.

TABLE 1. Coefficients for combined variable volume equations (K_1) and point sampling factor (K_2) for British Columbia tree species

Species	Region ^a	Maturity ^b	$K_1 \cdot 10^2$	K_2
Alder	C	M	0.230005	0.421706
Aspen	I	M	0.220876	0.404968
Balsam	C	M	0.236779	0.434126
"	I	M	0.222384	0.407733
Birch	I	M	0.202444	0.371173
Cedar	C	M	0.167661	0.307402
"	C	I	0.201329	0.369130
"	I	M	0.181726	0.333188
Cottonwood	C+I	M	0.203020	0.372230
Douglas-fir	C	M	0.167216	0.306585
"	C	I	0.202426	0.371142
"	I	M	0.181694	0.333130
Hemlock	C	M	0.208435	0.382158
"	C	I	0.210351	0.385671
"	I	M	0.222016	0.407058
Lodgepole pine	C+I	M	0.226124	0.414590
Larch	I	M	0.178894	0.327997
Maple	C	M	0.199804	0.366335
Spruce	C	M	0.190874	0.349961
"	I	M	0.211386	0.387568
White pine	C+I	M	0.224070	0.410825
Yellow cedar	C+I	M	0.219329	0.402132
Yellow pine	C+I	M	0.208576	0.382417

^aC is coast, I is interior

^bM is mature, I is immature

¹The assistance of Drs. D. D. Munro, J. H. G. Smith and A. Kozak is gratefully acknowledged.

TABLE 2. Comparison of general taper equations and localized taper equations for sample trees on the University Research Forest near Haney, British Columbia

Species	Source of equation ^a	K ₁ ·10 ²	K ₂ ^c	Bias of volume estimate ^b		
				(N + U)	(N)	(U)
Douglas-fir.....	URF Imm.	0.220401	0.404098	—	— 1.3 %	+36.4 %
	BCFS Imm.	0.202426	0.371142	— 8.2 %	— 9.4 %	+25.3 %
	BCFS Mat.	0.167216	0.306585	—24.1 %	—25.1 %	+ 3.5 %
Western hemlock.....	URF Imm.	0.239636	0.439364	—	—	—
	BCFS Imm.	0.210351	0.385671	—12.2 %	—	—
	BCFS Mat.	0.208435	0.382158	—13.0 %	—	—
Western red cedar.....	URF Imm.	0.218218	0.400107	—	— 3.5 %	+28.7 %
	BCFS Imm.	0.201329	0.369130	— 7.7 %	— 10.9 %	+18.7 %
	BCFS Mat.	0.167661	0.307402	—23.2 %	— 25.8 %	— 1.1 %

^aBCFS mature and immature equations were calculated by Kozak et al. (1969 b) from regional B.C. Forest Service taper curves. URF immature equations were calculated for the UBC Research Forest sample trees.

^b(N) is the bias when the taper equation is used for the Normal trees in the local sample, (U) for the Unusual shaped trees and (N + U) for both Normal and Unusual trees. There were 621 N and 19U Douglas-fir, 378 N western hemlock and no U hemlock, and 169 normal and 21 unusual shaped western red cedar trees.

^cThe differences in K₂ for (N + U) between URF and BCFS Imm. indicate a need for local adjustments since use of the BCFS Imm. equations could underestimate volumes by from 8% to 12%

The use of these factors for general inventory is not recommended, unless the bias of volume is tested and adjusted where necessary.

Recently, Smith and Kozak (1971) reported some results of comparisons of general taper equations and localized taper equations for sample trees on the University Research Forest near Haney, B.C. There was little improvement in diameter estimation (as measured by the standard error of estimate) by using taper equations developed for mature and immature trees, separately.

If taper equations are used, however, for the estimation of tree volume, application of separate equations for mature and immature trees can make a considerable difference as is shown in Table 2.

Unusually shaped Douglas-fir and western red cedar trees had K factors similar to the "mature" taper equations. For normally shaped trees important gain in similarity of the factors resulted from the use of locally derived taper equations. There are, of course, practical difficulties in field identification of individual trees likely to be fitted poorly by general taper equations.

Whenever taper equations are to be used to estimate tree volume, the standard error of estimate of section diameter is insufficient as a measure of precision, which should be calculated for tree volumes. With respect to the logarithmic volume and taper equation, a satisfactory method has been found to define the optimum equation for the estimation of both diameter and volume (Demaerschalk 1971). Work is now in progress to extend the same principle to other types of volume and taper equations.

COMMENTS, by F. EVERT, Forest Management Institute, Canadian Forestry Service, Ottawa.

If we multiply the product of tree basal area at breast height and tree height by a factor of 0.4, we get an approximate estimate of total tree volume under bark. For some species, this estimate tends to be low, e.g. red pine, and for other too high, e.g. cedars. For short trees, particularly up to 20 in height, the volume estimate obtained by factor 0.4

is low simply because of the arbitrarily chosen fixed height of measuring dbh.

To be able to obtain more precise estimate of volumes of standing trees than can be obtained by using the rule-of-thumb factor 0.4, a lot of work has been done since the publication of the first modern standard volume tables in 1817 by Cotta. And a lot of progress has been made, using multivariate functions in terms of dbh and height only, or using measures of upper-stem diameters as well.

Unfortunately, obtaining solutions to multivariate functions by regression analysis is not without pitfalls. One of these pitfalls is possible bias resulting from the preparation of tree-volume equations from data already expressed as equations, e.g. Smith and Bredon (1964) or as freehand curves, e.g. Kozak et al. (1964). This does not seem to have been recognized by Demaerschalk.

The British Columbia volume tables (Browne 1962) were derived from logarithmic tree-volume equations which were based on a total of 29,607 tree measurements of all species. The B.C. taper curves (Anon. 1962) were drawn by free-hand methods from the same measurements. Most of the B.C. logarithmic tree-volume equations for the total volume (v) and at least some B.C. taper curves show a decrease in tree breast-high form factor (f)¹ with increase in dbh (d) when tree height (h) stays constant, and an increase in the breast-high form factor with increase in the height when tree dbh stays constant.

¹Form factor is defined as the ratio between the volume of a tree and that of a cylinder having the same basal diameter and the same height.

References

- BROWNE, J. E. 1962. Standard cubic foot volume tables. BCFS Forest Surveys Invent. Div., Victoria, B.C., 107 p.
- DEMAERSCHALK, J. P. 1971. An integrated system for the estimation of tree taper and volume. Univ. B.C., Fac. Forest., M.F. thesis, 55 p.
- KOZAK, A., MUNRO, D. D. and J. H. G. SMITH. 1969a. Taper functions and their application in forest inventory. Forest. Chron. 45: 278-83.
- . 1969b. More accuracy required. Truck Logger. Dec. 20 - 1.
- SMITH, J. H. G. and D. D. MUNRO. 1965. Point sampling and merchantable volume factors for the commercial trees of British Columbia. Univ. B.C., Fac. Forest, Mimeo. 39 p.
- SMITH, J. H. G. and A. KOZAK. 1971. Further analyses of form and taper of young Douglas fir, western hemlock, western red cedar, and silver fir on the University of British Columbia Research Forest. Presented at Northwest Sci. Assoc. Ann. Meet., Univ. Idaho, April 16 - 17. 8 p.

Smith and Breadon (1964) converted the standard total-cubic-foot volume tables of the B.C. Forest Service to show total cubic foot volume in terms of d^2h . They found the correlation between Browne's volumes and those estimated by their equations impressively high. But, the combined variable equation:

$$v = a + b (d^2h/100)$$

when expressed as a form factor equation:

$$f = 183.3465 [a/(d^2h) + b(1/100)]$$

has the property that breast-high form factor will increase (if $a < 0$) or decrease (if $a > 0$) with increase in both the dbh and the height. Because of the differences between the form factor—dbh—height relationships in Browne's tables and those developed using the combined variable equation, the volume estimates obtained by the two methods also differ. Table 3 (Browne 1962) gives for 22-inch Interior Douglas-fir trees 40, 80, and 120 ft tall volumes of 29.4, 66.0, and 106 cu ft respectively, reflecting the increase in the breast-high form factor with tree height from 0.278 to 0.335. However, the combined variable equation (Smith and Breadon 1964) estimates the volumes of the same trees as 39.5, 69.5, and 99.5 cu ft, reflecting the decrease in the breast-high form factor with increasing tree height from 0.374 to 0.314.

Kozak et al. (1969) expressed the basic taper curves of the B.C. Forest Service (Anon. 1962) by a single taper function of each species which, when expressed as tree-volume equations by Demaerschalk, give a constant breast-high form factor for all trees of a given species regardless of their size. Because of the differences between the form factor—dbh—height relationships in the basic taper curves (Anon. 1962) and the constant form factor equations, the

volume estimates obtained by the two methods again differ. The volumes of a 100-ft Interior Douglas-fir — 10, 24, and 40 inches dbh — calculated from the basic taper curves, are 21.8, 102.4, and 264.8 cu ft, reflecting the decrease in breast-high form factor with dbh from 0.400 to 0.303. The constant form factor equation with a form factor of 0.33313 used by Demaerschalk estimates volumes of the same trees as 18.2, 104.7, and 290.7 cu ft respectively.

I am confident that the logarithmic equations (Browne 1962) and the taper curves (Anon. 1962) give a true picture of the breast-high form factor—dbh—height relationships of B.C. trees because they were prepared from measurements obtained on sectioned trees. Because the combined variable equations (Smith and Breadon 1964) and the taper functions (Kozak et al. 1964) do not necessarily reflect the relationships in the logarithmic equations and the taper curves, both the combined variable equations and the taper functions would therefore appear to give seriously biased estimates of B.C. tree volumes. Consequently, constant breast-high form factor equations are still only rule-of-thumb equations.

References

- ANON. 1962. Basic taper curves for the commercial soft-wood species of British Columbia. B.C. Forest Serv.
BROWNE, J.E. 1962. Cubic-foot volume tables for the commercial tree species of British Columbia. App. I, Table 3. B.C. Forest Serv.
KOZAK, A., D.D. MUNRO and J.H.G. SMITH. 1969. More accuracy required. *Truck Logger* 25(12):201-21.
SMITH, J.H.G. and R. E. BREADON. 1964. Combined variable equations and volume-basal area ratios for total cubic foot volumes of the commercial trees of B.C. Forest. *Chron.* 40:258-261.

REPLY, by J.P. DEMAERSCHALK

The comments by Mr. Evert could have been made more useful had he referred to some of his own excellent publications on the topic in the *Journal of Forestry*. These have made a fine case for use of form-height expressions in estimates of cruised volume and growth and yield from point sampling. Since only a few studies have been made in B.C. to illustrate the constants appropriate to application of his methods he ought to appreciate my attempt to secure approximations of form-heights. His warnings as to dangers in their use are quite appropriate for the most part but all too obvious to those with some experience in forest mensuration. Some of the limitations which concern him were made clear in my thesis. I have since elaborated on these and developed new taper equations which, when integrated, yield volume estimates identical to the logarithmic tree volume equations used by the B.C. Forest Service. A paper describing these is currently under review.

Nevertheless, he appears to have missed some important

implications of my article. These are the demonstration for the first time, I believe, of a simple method for conversion of taper equations to volume equations.

Obviously, freehand taper curves are of no practical use in electronic data processing and volume equations are of varying utility according to the objectives of the user.

Several studies at U.B.C. have shown that the logarithmic expressions of tree volume do not always provide the best estimates. Direct calculation of ratios of volume to basal area from basic data can be better. Further limitations exist in the fact that very few of the data used by the B.C. Forest Service have come from young stands and almost nothing is known about the response of young stands to management. Even first approximations of variable density yield equations are urgently needed in B.C. and can come from simple approaches such as have been described. The careful user as a result of Mr. Evert's comments will have been made more aware of possibilities of bias and, hopefully, will plan now to secure data and methods needed to improve the bases for his estimates of taper, volume, growth and yield.

FORESTAL
FORESTRY AND ENGINEERING INTERNATIONAL
LIMITED

FORESTAL
FORESTRY AND ENGINEERING
LIMITED

VANCOUVER 5, CANADA
1409 WEST PENDER STREET
Tel.: 683-6994 Cable: FORESTAL

CONSULTING
FORESTERS
ENGINEERS

**Industrial Forestry
Service Ltd.**

101-1595 FIFTH AVE.,
PRINCE GEORGE, B.C.
TEL.: 563-9207