Stand density management diagrams and their development and utility in black spruce management

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A stand density management diagram for black spruce (*Picea mariana* (Mill.) B.S.P.) was developed using data derived from 49 0.081-ha permanent sample plots and 257 open-grown sample trees located throughout central insular Newfoundland. The diagram illustrated the reciprocal equation of the competition-density effect, self-thinning rule, approximate crown closure line, zone of imminent competition-mortality, and isolines for relative density, quadratic mean diameter and merchantability ratio. Mean prediction error for natural stand trajectories over a 30-projection period were 2.5 dm³ for mean volume, 306 stems/ha for density, 16.1 m³/ha for merchantable volume, 14.3 m³/ha for total volume, and 1.9 m²/ha for basal area. Implementation procedures using a combination of monoareal and polyareal sampling methods were described and the potential application of the diagram for evaluating thinning alternatives was demonstrated. Limitations of the diagram and future research directions were identified.

Key words: stand density management diagram, black spruce, natural stands, central Newfoundland

Un diagramme d'aménagement en fonction de la densité du peuplement pour l'épinette noire (*Picea mariana* (Mill.) B.S.P.) a été élaboré à partir des données dérivées de 49 parcelles échantillons permanentes de 0.081 ha et de 257 arbres échantillons ayant poussé sans compétition dans la partie centrale de Terre-Neuve. Le diagramme illustre l'équation réciproque de l'effet de la compétition et de la densité, les règles d'auto-éclaircie, le niveau de fermeture approximative du couvert, la zone de mortalité imminente découlant de la compétition, et les isolignes de densité relative, de diamètre moyen obtenu par la moyenne quadratique et du ratio d'arbres de valeur marchande. L'erreur moyenne de prédiction pour l'évaluation de peuplements moyens au cours d'une période de 30 projections était de 2.5 dm³ pour le volume moyen, de 306 tiges/ha pour la densité, de 16.1 m³/ha pour le volume marchand, de 14.3 m³/ha pour le volume total et de 1.9 m²/ha pour la surface terrière. Les procédures d'implantation selon une combinaison de méthodes d'échantillonnage pour une population et pour plusieurs populations sont décrites et l'utilisation potentielle du diagramme pour l'évaluation des possibilités d'éclaircie est illustrée. Les limites du diagramme et l'orientation future des recherches sont identifiées.

Mots clés: diagramme d'aménagement en fonction de la densité du peuplement, épinette noire, peuplements naturels, centre de Terre-Neuve

Introduction

Stand density management is the process of controlling the level of growing stock through density manipulation in order to realize specified management objectives (Long 1985). Density manipulation consists of regulating the number and arrangement of individual stems per unit area through initial spacing or a temporal sequence of thinning events. Although the logistics of implementing density treatments are well developed, operational procedures for determining density control schedules by management objective are not available for many intensely managed boreal species. Consequently, the objectives of this study were to develop a stand density management diagram (SDMD) for black spruce (*Picea mariana* (Mill.) B.S.P.) and demonstrate its potential utility in density management.

Historical Development

SDMD's are average stand-level models which graphically illustrate the relationships between yield, density and mortality at various stages of stand development. They were initially developed by Japanese scientists for Hinoki cypress (*Chamaecyparis obtusa* (Siebold and Zucc.) Endl.), Japanese red pine (*Pinus densiflora* Siebold and Zucc.), Sugi

(*Cryptomeria japonica* D. Don.) and Japanese larch (*Larix leptolepis* (Siebold and Zucc.) Gord.) (Ando 1962, 1968; Tadaki 1963; Aiba 1975a,b). The reciprocal equation of the competition-density (C-D) effect (Eq. 1; Kira *et al.* 1953; Koyama and Kira 1956), and the $-3/2$ power law for self-thinning (Eq. 2; Yoda *et al.* 1963) were the principal relationships employed.

$$1/\bar{v} = k_1 + k_2 \cdot N \quad (1)$$

$$\bar{v} = k_3 \cdot N^{-3/2} \quad (2)$$

where $\bar{v}$ is the mean stem volume per tree, N is the number of trees per unit area, k_1 is a species-specific constant dependent on the stage of stand development, k_2 is a constant ranging in value from zero to unity depending on the intensity of competition, and k_3 is a constant of proportionality.

Drew and Flewelling (1977, 1979) introduced SDMD's to the North American forestry literature by reviewing their underlying relationships and developing diagrams for Monterey pine (*Pinus radiata* D. Don.) and coastal Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco). They formalized the phase of stand development between the initiation of density-dependent mortality and the asymptotic $\bar{v}$ - N relationship during self-thinning (the $-3/2$ power law) as the zone of imminent competition-mortality. Furthermore, they quantified the relative intensity of competition as the ratio of the observed stand density to the maximum stand density attainable in a stand with the same mean tree volume

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(relative density index (P_r)). Employing this framework they also developed SDMD's for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) and lodgepole pine (*Pinus contorta* Dougl.) (Flewelling *et al.* 1980, 1985).

McCarter and Long (1986) calibrated a SDMD for lodgepole pine but employed Reineke's (1933) stand density index rather than relative density index. In addition, they used mean diameter as the size variate rather than mean volume. The rationale for these changes was to reduce the complexities of estimating mean volume when implementing SDMD's in the field. Smith and Long (1987) extended McCarter and Long's (1986) diagram to wildlife habitat evaluation by delineating relative density zones conducive to elk (*Cervus elaphus nelsonii*) and mule deer (*Odocoileus hemionus hemionus*) habitat (i.e. hiding and thermal cover requirements). Furthermore, the consequences of various thinning treatments on elk and deer habitat were simulated using the diagram. Smith (1989) developed a SDMD for western red cedar (*Thuja plicata* Donn ex D. Don) based on a system of temporal-dependent equations (e.g., mean volume was modeled as a function of density (Smith and Hann 1986)). In addition to density management, Smith (1989) discussed the potential of using the diagram for vegetation management: deriving initial spacings which would limit salal (*Gaultheria shallon* Pursh.) development.

Method

The approach employed in the development of a SDMD for black spruce consisted of the following procedures: (1) determining the mean volume-density relationship and relative density index (Drew and Flewelling 1979) corresponding to (a) the asymptotic mean volume-density condition ($-3/2$ power law for self-thinning (Yoda *et al.* 1963)), (b) the lower limit of the imminent competition-mortality zone (Drew and Flewelling 1979), and (c) approximate crown closure (2) calibrating the reciprocal equation of the C-D effect at specific stages of stand development (as defined by dominant height class) and (3) deriving quadratic mean diameter and merchantable volume/total volume ratio isolines.

Study Area

Forest Section B28a-Grand Falls of the Canadian boreal forest region (Rowe 1972) occupies the plateau of central insular Newfoundland. The forest vegetation is dominated by black spruce and balsam fir (*Abies balsamea* (L.) Mill.) with eastern larch (*Larix laricina* (Du Roi) K. Koch.), white spruce (*Picea glauca* (Moench) Voss), trembling aspen (*Populus tremuloides* Michx.) and white birch (*Betula papyrifera* Marsh.) scattered throughout. Commonly occurring boreal shrub, herb, moss and lichen species are widely distributed within the study area (e.g., low-bush blueberry (*Vaccinium angustifolium* Ait.), bunch-berry (*Cornus canadensis* L.), *Pleurozium schreberi* (BSG.) Mitt.) and reindeer moss (*Cladonia rangiferina* (L.) Web.). The area's undulating topography ranges in elevation from 75 to 200 m above mean sea level. Throughout most of the area, shale, slate, sandstone and schists underlie a stony glaciated till (Damman 1964). Well-drained Podzol soils occur on upland sites whereas poorly-drained gleysols and organic soils are found on lowland areas. The humid climate is characterized by moderate summer and winter temperatures. The growing season is approximately 150 days in length, beginning in late

May and ending in early October (Damman 1983). Mean number of degree days ($>5^{\circ}\text{C}$) and precipitation during the vegetative season are approximately 1170 and 430 mm, respectively (AES 1970-86).

Data Base

The data base consisted of 49 0.081-ha permanent sample plots (PSP's) and 257 open-grown sample trees located in naturally regenerated black spruce stands ($>90\%$ black spruce by basal area) throughout the study area. The PSP's were established in 1947-48 and remeasured from one to three times at approximately 10 year intervals. Specifically, all 49 PSP's were remeasured once (1957-58), 47 remeasured twice (1957-58/1967-68) and 30 remeasured three times (1957-58/1967-68/1978), yielding a total of 175 measurements. Data collected at establishment and subsequent remeasurements consisted of (1) a grouped diameter frequency distribution tally for all living trees greater than 1.5 cm breast-height diameter outside-bark (DBHOB), by species and 2.5 cm diameter classes and (2) DBHOB (± 0.03 cm), total height (± 0.31 m) and breast-height age (AGE_{bh}) measurements on approximately ten dominant and (or) codominant black spruce trees. During the 1957-58 remeasurement and subsequent remeasurements dead trees were marked and tallied by diameter class and species. For each plot measurement, data computations consisted of the following procedures: (1) calculating heights for each diameter class using Page and van Nostrand's (1973) height-diameter regression functions (i.e., $H_{t(i)} = 12.649 + 0.1045 \cdot D_{(i)} \cdot HT_{d(j)} - 0.0144 \cdot D_{(i)}^2$ where $H_{t(i)}$ is the height of the i th diameter class in feet, $D_{(i)}$ is the diameter class mid-point value in inches, and $HT_{d(j)}$ is the mean dominant plot height in feet at the j th measurement) (2) transforming diameter and height values to their metric equivalents employing Ker's (1974) procedure and (3) computing total and merchantable volumes using Warren and Meades' (1986) equations. Merchantability limits consisted of a 0.1524 m stump-height and a 7.62 cm top diameter (inside-bark), for all stems greater than 9.5 cm in diameter (DBHOB). All other plot measurements were converted to their metric

Table 1. Mensurational characteristics of the data employed

Parameter	Mean	SD	Range
Permanent sample plot measurements ($n=175$)			
DBHOB ¹ (cm)	7.4	1.80	3.3-14.7
HT _d (m)	10.2	2.26	4.5-15.8
AGE _{bh} ² (yr)	45.4	12.35	14-78
Site index ³	10.8	1.38	8.1-13.4
Mean volume (dm ³)	18.2	13.41	2.02-98.2
Density (stems/ha)	6680.6	3170.20	1161-19410
Merch. volume ⁴ (dm ³ /ha)	72755.6	55438.89	0.0-281394.0
Total volume (dm ³ /ha)	106290.6	56970.33	4626.4-305188.4
Basal area (dm ² /ha)	2767.7	1177.55	197.7-5546.4
Open grown sample trees ($n=257$)			
DBHOB (cm)	14.3	1.43	5.9-30.7
HT (m)	7.5	1.36	3.9-14.0
CW (m)	3.4	1.28	1.9-6.9

¹Quadratic mean diameter outside-bark.

²Dominant breast-height age.

³Mean dominant height (m) at a breast-height age of 50 years (Newton 1992).

⁴Merchantable volume: 0.1524 m stump-height and 7.62 cm top diameter (inside-bark) for all trees greater than 9.5 cm in diameter at breast-height.

equivalents using standard conversion factors (System of International Units). Each plot was assigned a site index value (SI) based on the mean dominant height (HT_d) and AGE_{bh} measurements taken during the 1957–58 remeasurement (i.e., HT_d at an AGE_{bh} of 50 years). Site index values were obtained from Newton's (1992) base-age invariant polymorphic site index curves developed for black spruce in central insular Newfoundland.

Individual black spruce sample trees characterized by live-crown ratios of approximately 95% were selected within open stands throughout the study area. Measurements on each sample tree consisted of DBHOB (± 0.1 cm), height (HT; ± 0.3 m) and mean crown-width (CW; ± 0.1 m; computed from one radii per quadrant). Allometric equations for the HT-DBHOB and CW-DBHOB relationships were parameterized using ordinary least squares (OLS) regression procedures on logarithmically transformed data (Eqs. 3 and 4).

$$HT = 0.974 \cdot DBHOB^{0.769} \quad (3)$$

$$CW = 0.870 \cdot DBHOB^{0.513} \quad (4)$$

Associated regression statistics for Eq. 3 were as follows: coefficient of determination (r^2) = 0.802; standard error of the estimate (SEE; $\log_e$) = 0.1383; F -ratio = 1031.1 ($P \leq 0.05$); and $n = 257$. Similarly, for Eq. 4: r^2 = 0.576; SEE ($\log_e$) = 0.1592; F -ratio = 345.9 ($P \leq 0.05$); and $n = 257$. Table 1 summarizes the mensurational characteristics of the data employed.

Data Analysis

Self-thinning Rule. Employing volume-density data derived from self-thinning black spruce stands ($n = 34$) within the study area, Newton and Smith (1990) estimated the parameters of the $-3/2$ power law for self-thinning. Specifically, the analytical procedures proposed by Weller (1987) were employed: relating total volume per unit area (V) to number of trees per unit area using principal component analysis and interpreting the resultant relationship through tests of hypotheses. The analysis indicated that V and N were negatively correlated ($r = -0.862$; $P \leq 0.05$) with a thinning exponent of -0.618 . Transforming the parameter estimates to a mean volume (dm^3) – density (stems/ha) basis, the self-thinning relationship was obtained (Eq. 5).

$$\bar{v} = 10^{7.691} \cdot N^{-1.618} \quad (5)$$

The equation for predicting relative densities (Drew and Flewelling 1979) for observed mean volumes and densities (N_o), was derived from the self-thinning relationship (Eq. 6).

$$P_r = N_o / (\bar{v} / 10^{7.691})^{-0.618} \quad (6)$$

Lower Limit of the Imminent Competition-Mortality Zone. Drew and Flewelling (1979) defined the zone of imminent competition-mortality as the stage of stand development in which density-dependent mortality was likely to occur. Employing this criteria mortality data were derived from the PSPs and analyzed in reference to its rate (percentage of stems that died during each 10 year measurement interval) and distribution (positive skewness of the diameter distribution of dead stems was considered indicative of stands incurring density-dependent mortality (Weiner 1988)). The PSP data were stratified into relative density classes (0.10 class widths) and mean mortality rates, and corresponding 95%

confidence intervals (CI_{95}), calculated. Mortality rates (%/decade) ranged from a minimum of zero for P_r classes 0.05 and 0.15 to a maximum of 12% for the P_r class of 1.15. Relative densities exceeded unity for some plots given that the $-3/2$ power law for self-thinning represents the average asymptotic volume-density condition within self-thinning stands. Relative density values corresponding to mean mortality rates significantly greater than zero were considered to be within the zone of imminent competition-mortality (i.e., $CI_{L95} > 0.0$ where CI_{L95} is the lower boundary of the 95% confidence interval). Hence, the relative density corresponding to the lower limit of this zone was defined as the initial value at which the mean mortality rates significantly differed from zero. Employing this procedure, the lower limit of the zone of imminent competition-mortality was delineated by a P_r of 0.50.

Approximate Crown Closure Line. The P_r isoline corresponding to approximate crown closure was determined by the following method. Employing the range of diameter values observed within the PSP data, volumes were estimated by 1 cm diameter classes (3.0–15.0 cm) using Warren and Meades' (1986) volume equation in combination with the HT-DBHOB allometric relationship for open-grown trees (Eq. 3). Corresponding densities for each diameter class were estimated by dividing the projected crown area (CA) into ten-thousand and multiplying by a spatial pattern conversion factor: $N_{cc} = (10000/CA) \cdot 0.7854$ where N_{cc} is the density (stems/ha) at crown closure for a given DBHOB, 10000 is the area per hectare in square meters, $CA = \pi \cdot (CW/2)^2$ where CW was estimated using the DBHOB-CW allometric relationship for open-grown trees (Eq. 4), and 0.7854 is a conversion factor (Smith 1989). Given the estimated stem volumes and densities at crown closure the mean P_r was computed: mean $P_r = 0.13$ (SD = 0.04). Consequently, crown closure was assumed to occur at a relative density of 0.13.

Reciprocal Equation of the C-D Effect by HT_d Class. Reciprocal relationships for the C-D effect were derived using a two-stage regression modelling procedure. In the first-stage, parameter estimates for the reciprocal equation were obtained by HT_d class. The second-stage consisted of relating the first-stage parameter estimates to HT_d . First-stage estimation error and second-stage error term correlation were assumed to be zero. Specifically, the method consisted of stratifying the PSP $\bar{v}$ - N data into HT_d classes (6.0–15.0 by 1-m intervals) and fitting Eq. 1 using OLS regression procedures. Results indicated that except for two HT_d classes the relationships were significant ($P \leq 0.05$) (Table 2). The first-stage parameter estimates were then related to HT_d employing Eqs. 7 and 8.

$$\log_{10}(\hat{k}_1) = b_0 + b_1 \cdot HT_d \quad (7)$$

$$\log_{10}(\hat{k}_2) = b_2 + b_3 \cdot HT_d \quad (8)$$

where b_i , $i=0,4$ are parameter estimates obtained by OLS regression techniques. Resultant parameter estimates and associated regression statistics for Eq. 7 were as follows: $b_0 = 0.2741$; $b_1 = -0.1803$; $r^2 = 0.948$; SEE ($\log_{10}$) = 0.1137; F -ratio = 90.4 ($P \leq 0.05$); and $n = 7$. Similarly, for Eq. 8: $b_2 = -4.9861$; $b_3 = -0.0251$; $r^2 = 0.866$; SEE ($\log_{10}$) = 0.0265; F -ratio = 32.3 ($P \leq 0.05$); and

Table 2. Parameter estimates and associated regression statistics for the reciprocal equation of the C-D effect (Eq. 1): first-stage results

HT _d class ¹ (m)	Parameter estimates		n	Regression statistics ²		
	$\hat{k}_1$	$\hat{k}_2$		SEE (1/dm ³)	r ²	F-ratio
6.5	—	—	11	—	—	ns
7.5	0.0678	$0.7063 \cdot 10^{-5}$	18	0.0366	0.465	13.9*
8.5	—	—	20	—	—	ns
9.5	0.0456	$0.5918 \cdot 10^{-5}$	32	0.0239	0.379	18.3*
10.5	0.0233	$0.5363 \cdot 10^{-5}$	24	0.0100	0.684	47.7*
11.5	0.0158	$0.5234 \cdot 10^{-5}$	29	0.0110	0.689	59.9*
12.5	0.0162	$0.4626 \cdot 10^{-5}$	12	0.0126	0.477	14.6*
13.5	0.0058	$0.5111 \cdot 10^{-5}$	9	0.0095	0.566	13.1*
14.5	0.0036	$0.4536 \cdot 10^{-5}$	18	0.0100	0.540	8.2*

¹Dominant height (mid-point value).

²Standard error of estimate (SEE); coefficient of determination (r²); and F-ratio for the overall regression relationship where ns and superscript * indicate non-significant (P > 0.05) and significant (P ≤ 0.05) relationships, respectively.

n = 7. Second-stage models were then incorporated back into the first-stage models and parameter estimates predicted for specific HT_d class values (3.0–14.0 m by 1-m intervals).

Diameter and Merchantable Volume/Total Volume Ratio Isolines. Quadratic mean diameter (DBHOB_q) was related to mean volume and density employing the Eq. 9.

$$\log_{10}(\text{DBHOB}_q) = b_4 + b_5 \cdot \log_{10}(\bar{v}) + b_6 \cdot \log_{10}(N) \quad (9)$$

where $b_i, i=4,6$ are parameter estimates obtained by OLS regression techniques on logarithmically transformed data. Resultant parameter estimates and associated regression statistics were as follows: $b_4 = 0.5183$; $b_5 = 0.3625$; $b_6 = -0.0221$; multiple coefficient of determination (R^2) = 0.986; SEE ($\log_{10}$) = 0.0118; F-ratio = 6206.9 ($P \leq 0.05$); and $n = 175$. Isolines were obtained by rearranging the function with respect to mean volume and specifying diameter values. The merchantable volume/total volume ratio (MV/TV) was related to mean volume using Eq. 10.

$$\log_e(1.0 - \text{MV/TV}) = b_7 + b_8 \cdot \bar{v} + b_9 \cdot \bar{v}^2 \quad (10)$$

where $b_i, i=7,9$ are parameter estimates obtained by OLS regression techniques. Resultant parameter estimates and associated regression statistics were as follows: $b_7 = 0.0805$;

Figure 1. Black spruce stand density management diagram graphically illustrating: (1) self-thinning rule at a relative density index (Pr) of 1.0 (2) lower limit of the zone of imminent competition-mortality (ZICM) at Pr = 0.50 (3) approximate crown closure line (CC) at Pr = 0.13 (4) reciprocal equation of the C-D effect at specified dominant heights (HT isolines) and (5) quadratic mean diameter (DBHOB) and merchantable volume/total volume ratio (MV/TV) isolines.

$b_8 = -0.0790$; $b_9 = 0.0004$; $R^2 = 0.952$; $SEE(\log_e) = 0.1567$; $F\text{-ratio} = 1701.0$ ($P \leq 0.05$); and $n = 175$. Merchantable volume/total volume ratio isolines were obtained by rearranging the equation into a quadratic form ($a \cdot \bar{v}^2 + b \cdot \bar{v} + c = 0$ where $a = b_9$, $b = b_8$ and $c = b_7 - \log_e(1.0 - MV/TV)$) and obtaining the roots ($x = (-b \pm (b^2 - 4ac)^{1/2})/2a$) at specified MV/TV ratios (0.10–0.90 by 0.10 increments and at 0.95). The resultant roots indicate the mean volume values corresponding to the specified MV/TV ratios.

Results

Superimposing the reciprocal equation of the C-D effect, self-thinning rule, lower limit of the zone of imminent competition-mortality, approximate crown closure line, and isolines for relative density index, quadratic mean diameter and merchantable volume/total volume ratio, on a bivariate graph with mean volume on the ordinate axis and stand density on the abscissa, a SDMD for black spruce was developed (Fig. 1). The diagram was evaluated by (1) graphically comparing the PSPs mean volume-density trajectories with those described by the diagram, and (2) quantitatively

comparing the PSPs mean volume, density, merchantable volume, total volume, and total basal area values, with those predicted by the diagram.

Graphical analysis indicated that most PSPs illustrated a considerable degree of ingrowth at P_r s below 0.50 (i.e., trees moving into the lowest measurable size class ($1.5 < DBHOB \text{ (cm)} < 4.1$)). At P_r s greater than 0.50 most PSPs exhibited a reduction in density (i.e., zone of imminent competition-mortality). Furthermore, for $P_r \geq 0.80$ the trajectories paralleled the asymptotic $\bar{v}$ - N relationship. These trajectories closely corresponded with those predicted by the diagram: stands within the zone of imminent competition-mortality will incur density-dependent mortality, and stands at a maximum $\bar{v}$ - N condition will thin along a slope of approximately -1.6 .

In order to account for increases in density due to ingrowth and differential rates of mortality within the zone of imminent competition-mortality the following empirical equations were developed for use when employing the SDMD for describing natural stand trajectories: (1) $IGR = -33.1444 + 296.2085 \cdot (1.0/HT_d) + 8.2261 \cdot (1.0/P_r)$ where IGR is

Figure 2. Black spruce stand density management diagram with estimated natural stand trajectories superimposed by site index class. (a) SI = 9: trajectories for specified densities from an AGE_{bh} of 25 years to an AGE_{bh} of 75 years. (b) SI = 11: trajectories for specified densities from an AGE_{bh} of 20 years to an AGE_{bh} of 70 years. (c) SI = 13: trajectories for specified densities from an AGE_{bh} of 15 years to an AGE_{bh} of 65 years. Note, 10 year intervals denoted by open squares.

Fig. 2b

the mean percentage of the number of trees becoming measurable during a ten year period relative to the initial number of trees measurable at the beginning of the period, for stands at P_r s below 0.50; and (2) $MR = 0.777051 \cdot e^{2.169675 \cdot P_r}$ where MR is the mean percentage of the number of trees that die during a 10-year period relative to the initial number of trees at the beginning of the period for stands within the zone of imminent competition-mortality. Figure 2 graphically illustrates the expected natural stand trajectories by site index class.

Employing these ingrowth and mortality rate functions, the PSPs data were compared with that predicted by the diagram using Reynold's (1984) error estimation procedure for all plots with three remeasurements ($n = 30$). Results indicated mean prediction errors over a 30-year projection period of 2.5 dm³ for mean volume, 306 stems/ha for density, 16058.2 dm³/ha for merchantable volume, 14268.6 dm³/ha for total volume, and 186.1 dm²/ha for basal area (Table 3). However, due to the lack of an independent test data set these results should be considered tentative.

Discussion

The SDMD developed for black spruce graphically illustrates the relationships between yield, density and mortality at various stages of stand development. The diagram can be

used to describe the general developmental patterns of natural black spruce stands. Subject stands can be superimposed on the diagram and their expected natural thinning trajectories derived. Furthermore, future estimates of mean volumes, densities, quadratic mean diameters, merchantable and total volumes, and total basal areas can also be obtained using the dominant height isolines. Thinning scenarios can be superimposed on the diagram if the type of thinning treatment approximates the natural thinning process (e.g., randomly removing the smaller sized individuals from spatially aggregated clumps).

Implementation Procedures

In order to apply the diagram in the field, the user would locate the subject stand on the diagram by determining its density, mean diameter and dominant height (n , mean diameter has been used as a surrogate location measure given the complexities of estimating mean stem volume). Superimposing these points on the diagram, approximate intersection should occur (e.g., N-DBHOB_q point should intersect the specified HT_d line). In cases in which intersection does not occur, extraneous factors may have influenced either diameter or height growth (e.g., partial thinning or spruce budworm (*Choristoneura fumiferana* (Clem)) leader damage). Thus in situations where diameter growth has been

Fig. 2c

affected, the $N\text{-}HT_d$ coordinate should be used to position the subject stand on the diagram. Conversely, if height growth patterns were affected, then the $N\text{-}DBHOB_q$ coordinate should be used. Once located on the dominant height line, future projections could be derived using the estimated dominant height development as described by the site index curves. Actual determination of site index, density, mean diameter and dominant height of the subject stand can be readily accomplished by combining monoareal (fixed-size plots) and polyareal (variable-size plots) sampling procedures (Husch *et al.* 1982).

Specifically, estimates of site index, dominant height and density could be obtained from the fixed-size plot measurements whereas horizontal point-sampling could provide an estimate of the quadratic mean diameter. For example, temporary fixed-size circular plots could be established throughout the subject stand, and the following measurements taken per plot (e.g., three 5-m radius plots).

(1) HT_d and AGE_{bh} would be determined from a subsample of dominant trees ($n = 10$ trees).

(2) Using HT_d and AGE_{bh} , SI would be obtained from the site index curves (Newton 1992).

(3) Stand density would be estimated by converting the observed density/plot to a per-hectare basis.

(4) At the centre of the fixed-size plot, an optical wedge

prism sweep would be taken and the number of trees selected tallied by diameter class (N_i = number of trees in the i th diameter class ($i = 1, \dots, n$)).

(5) Using the tree factor (TF) for each diameter class, total density estimates (N_e) would be obtained ($N_e = \sum_{i=1}^n (TF \cdot N_i)$ where $TF = BAF / (C \cdot DBHOB^2)$ (BAF is the basal area factor of the prism) and C is a constant (e.g., $C = 0.00007854$ in metric units)).

(6) Dividing the total basal area ($(\sum_{i=1}^n N_i) \cdot BAF$) by the estimated total density (N_e) would yield the mean basal area per tree (BA_m).

(7) Dividing BA_m by C and obtaining the square root, would give the quadratic mean diameter ($DBHOB_q = ((BA_m/C)^{1/2})$).

Using the average site index, density, mean diameter and dominant height, calculated from the three sample plots, the subject stand can be positioned on the diagram and consequences of various density manipulations evaluated.

Potential Utility

The utility of the diagram can be demonstrated by considering the following simulation. Specifically, these simulations illustrate the potential of employing thinning treatments within dense black spruce stands in order to obtain merchantable fibre in the short-term, while still achieving comparable yields at rotation age.

Table 3. Predictive ability of the black spruce stand density management diagram: approximate 95% confidence intervals for prediction error

Parameter	Projection period (yrs)	Distribution of errors K-S statistic ¹	Prediction error ²	
			Mean	CI ₉₅
Mean volume (dm ³)	30	0.332*	2.50 ^{ns}	-2.25-7.25
Total density (stems/ha)	30	0.137 ^{ns}	306.94 ^{ns}	-356.98-970.86
Merchantable volume (dm ³ /ha)	30	0.116 ^{ns}	16058.21*	2863.35-29253.07
Total volume (dm ³ /ha)	30	0.149 ^{ns}	14268.64*	814.60-27722.68
Basal area (dm ² /ha)	30	0.110 ^{ns}	186.12 ^{ns}	-78.89-451.14

¹Kolmogorov-Smirnov goodness of fit test statistic for testing the null hypothesis that the distribution of errors are normally distributed (i.e., difference (D_i) = observed - predicted for the i th plot ($i = 1, \dots, n$; $n = 30$)); Superscripts ns and * refer to the acceptance and rejection of the null hypothesis at the 0.05 probability level, respectively.

²Mean error ($\bar{D} = \sum_{i=1}^n D_i/n$) and 95% confidence intervals for $\bar{D}$; Superscripts ns and * refer to the acceptance and rejection of the null hypothesis that the predictions are not biased at the 0.05 probability level, respectively (i.e., $H_0: \bar{D} = 0$); 95% CI for $E(\bar{D})$ was calculated as follows: $\bar{D} \pm S \cdot t_{(1-\alpha/2; n-1)} / n^{1/2}$ where $S = (\sum_{i=1}^n (D_i - \bar{D})^2 / (n-1))^{1/2}$ and $t_{(1-\alpha/2; n-1)}$ is the $1 - \alpha/2$ quantile of the Student- t distribution with $n - 1$ degrees of freedom (Reynolds 1984).

Figure 3. Thinning simulations superimposed on the black spruce stand density management diagram for site index 13. Labels T-15, T-25 and T-35 refer to thinning treatments at breast-height ages of 15, 25 and 35, respectively (denoted by the horizontal curved lines (---)). The vertical straight line on the left illustrates the volume-density trajectory for thinned stands by 10 year intervals (15-65 years as denoted by open squares). The vertical curve line on the right illustrates the volume-density trajectory for the control stand by 10 year intervals (15-65 years as denoted by open squares).

Given a 15-year-old (AGE_{bh}) subject stand at the lower limit of the zone of imminent competition-mortality, the management objective is to control stand density in such a way as to avoid density-dependent mortality (i.e., achieving a P_r of 0.50 at a rotation age of 65 years (AGE_{bh})). The site

index of the subject stand is 13. Four treatment options are available: (1) do not thin (CONTROL); (2) thin at 15 years (T-15); (3) thin at 25 years (T-25); or (4) thin at 35 years (T-35). Assuming that the type of thinning is random with a bias towards the smaller size trees situated within spatially

Table 4. Example simulations employing the stand density management diagram for black spruce: site index = 13 m (Newton 1992)

Treatment ¹ (stems/ha) [AGE _{bh} (yr)]			$\bar{v}_{65}$ ² (dm ³)	DBHOB _q ³ (cm)	Merchantable yield ⁴ (dm ³ /ha)				
					Final (1)	Thinning (2)	Total production (1 + 2)	Difference ⁵ (dm ³ /ha)	(%)
Control	[15]	10000 → 7440	24.5	8.6	142079.2	–	142079.2	–	–
T-15	[15]	10000 → 2250	59.7	12.2	127882.1	8668.4	136550.5	–5528.7	–3.9
T-25	[25]	9579 → 2250	59.7	12.2	127882.1	29250.1	157132.2	15053.0	10.6
T-35	[35]	9056 → 2250	59.7	12.2	127882.1	37843.0	165725.1	23645.9	16.6

¹Simulated treatments as graphically illustrated on Fig. 3; The control stand is at the lower limit of the zone of imminent competition-mortality, AGE_{bh} is 15 years and the stand is assumed to follow a natural $\bar{v}$ -N trajectory until an AGE_{bh} of 65 years; Treatments consisted of a random thinning in which N was reduced to a target density of 2250 stems/ha; Note, this target N reflects a stand developmental pattern in which density-dependent mortality is approximately nil.

²Projected $\bar{v}$ at an AGE_{bh} of 65 years.

³Projected DBHOB_q at an AGE_{bh} of 65 years.

⁴Projected final yields at an AGE_{bh} of 65 years, thinning yields at the time treatment, and total production at an AGE_{bh} of 65 years.

⁵Merchantable yield differences between control and treated stands.

aggregated clumps, and the thinning operation would not change the underlying spatial pattern, the SDMD could be used.

From the diagram the target density is derived (2250 stems/ha): density corresponding to the point of intersection between the HT_d isoline at age 65 on a site quality of 13 (e.g., HT_d = 13.8) and the P_r = 0.50 isoline (Fig. 3). The four treatment scenarios are superimposed on the diagram (Fig. 3). Mean diameter and volume at 65 years are obtained using the isolines (diameter isolines intersecting the HT_d isoline at the treatment densities). Merchantable volumes are obtained by multiplying the total volume (TV = $\bar{v} \cdot N$) by the MV/TV ratio isoline value. Merchantable yields from thinning (MV_t) are estimated by determining the total volume removed and multiplying by the MV/TV ratio: $MV_t = (\bar{v}_{ct} \cdot N_{ct} - \bar{v}_{tt} \cdot N_{tt}) \cdot MV/TV_{ct}$ where $\bar{v}_{ct}$ and N_{ct} are the mean volume and density of the control stand at time t , $\bar{v}_{tt}$ and N_{tt} are the mean volume and density of the thinned stand at time t , and MV/TV_{ct} is the merchantable volume/total volume ratio of the control stand at time t . Table 4 lists the resultant yields of these simulations.

Thinning treatments increased quadratic mean diameter and volume over the control stand by 71% and 144%, respectively. Total merchantable yields were greater for the thinned stands than the unthinned stands for T-25 and T-35 treatments. Timing of treatments also influenced yields: the later thinning treatments yielded greater merchantable volumes than earlier treatments. The results of these simulations illustrate the potential application of stand density management diagrams in density management decision-making. However, since the diagram is based on unthinned stand dynamics, representation of the effects of thinning may not fully reflect the true response. Consequently, explicit interpretation of thinning responses should be avoided until further evaluations are conducted (e.g., evaluations employing data from thinned stands).

Further Research

The applicability of the diagram developed in this study for describing the development of thinned stands will be largely unknown until experimental results derived from various trials become available. Furthermore, a separate diagram employing data derived from black spruce plantations should be developed (n , such a diagram is currently being calibrated). Isolines for additional yield components must be defined before the full potential of the diagram is realized

(e.g., form factors and wood density). Furthermore, stand density conditions conducive to *Kalmia angustifolia* (L.) growth and development should be identified and delineated on the diagram (i.e., *Kalmia angustifolia* (L.) is an ericaceous shrub species commonly associated with black spruce within central-insular Newfoundland and is considered allelopathic to black spruce (Mallik 1987)).

Conclusions

SDMD's are biologically based models which allow forest managers to formulate reasonable consequences to various density manipulations. They are particularly useful in situations where density management decisions must be made in the absence of information on managed stands. Furthermore, SDMD's are very useful for crop planning in the field because the natural stand dynamics are presented in a single two-dimensional diagram. A forester can readily see at what heights and diameters black spruce stands start to close and subsequently incur mortality. The consequences of various thinning scenarios can be plotted and visualized in the field without the need for computer simulation. In this respect they are very useful for teaching and understanding. The black spruce diagram developed in this study should be considered as a first-approximation. Further research is required before the full potential of these diagrams is realized (e.g., defining isolines for other yield components and delineating density conditions conducive to *Kalmia angustifolia* (L.) growth).

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