Variable stocking yield functions for the boreal mixedwood in Ontario

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A measure of utilized site productivity, "basal area index" was recently developed and used to construct variable stocking yield functions and tables for the boreal mixedwood of northcentral Ontario. Data from 197 permanent sample plots supplied by the James River/Marathon Paper Company Ltd. were used. The resulting yield equations compare favourably with previous ones and are more appropriate for the boreal mixedwood. In addition, the basal area index employed has several advantages: 1) it serves as a valid measure of utilized site productivity which is better correlated with the main stand attributes than site index; 2) it produces variable stocking yield tables suitable for uneven-aged mixed species cover types; 3) unlike the site index, the basal area index may be estimated quickly, easily and inexpensively. The basic improvement in mixedwood yield estimation via basal area index should have broad applications for other stand types particularly for the disturbed hardwoods of southern Ontario and those in eastern United States.

Key words: Basal area index, site index model, numerical method

Une mesure de la productivité utilisée d'une station, un "indice de la surface terrière", a été élaborée et utilisée pour établir des fonctions et des tables de rendement selon une densité variable pour les peuplements mélangés nordiques du centre-nord de l'Ontario. Les données de 197 parcelles-échantillons permanentes fournies par la compagnie James River/Marathon Paper ont été analysées. Les équations de rendement obtenues se comparent adéquatement aux équations obtenues précédemment et sont plus appropriées aux peuplements mélangés nordiques. De plus, l'indice de surface terrière utilisé comportait certains avantages: 1) il servait de mesure valide de la productivité utilisée d'une station qui était mieux corrélée aux caractéristiques principales du peuplement que ne l'est l'indice de station; 2) il permettait d'obtenir des tables de rendement selon une densité variable utilisables pour les peuplements inéquiennes d'espèces mélangées; 3) contrairement à l'indice de station, l'indice de la surface terrière peut être évalué rapidement, facilement et à peu de frais. L'amélioration fondamentale de l'estimation du rendement des peuplements mélangés au moyen de l'indice de la surface terrière devrait avoir des applications générales pour les autres types de peuplements, en particulier dans le cas de peuplements feuillus perturbés du sud de l'Ontario et ceux de l'est des États-Unis.

Mots clés: indice de la surface terrière, modèle de l'indice de la station, méthode numérique

Introduction

Nearly 3 million ha of Ontario's forest lands are classified as boreal mixedwood of black spruce (*Picea mariana* [Mill] B.S.P.), white spruce (*Picea glauca* [Moench] Voss), Jack pine (*Pinus banksiana* Lamb.), poplar (*Populus balsamifera*) and aspen (*Populus tremuloides* Michx.) cover types. Because of wood inventories and the increased demand for both wood and recreational use of forest lands, greater demand is being placed on the boreal mixedwood forests of Ontario.

Managing such complicated forest types requires extensive knowledge of their extent, composition, and growth and yield. Forest managers require detailed and up-to-date information on stand productivity under various management intensities. Although several major studies were undertaken on the boreal mixedwood during the past 50 years, little yield information was published.

The first cooperative yield study of northwestern Ontario's mixedwood began in 1948. Several pulp and paper companies jointly requested the Canadian Forest Service to provide yield information on the boreal mixedwood in the Nipigon district (Bedell and MacLean 1952). The companies agreed to collect the required data while the Ministry assumed responsibility

for data compilation and analysis. In 1974 it was further agreed that the Petawawa National Forestry Institute would prepare separate inventory-oriented stand development tables for American Can of Canada Ltd. and Kimberly-Clark of Canada Ltd., then Ontario Region would process the data to provide further yield information. Evert (1975) published separate stand development curves and tables for each company by forest type, but without stratification by site. This useful report was well received by both companies involved and thus satisfied the primary objective of the above-mentioned agreement. Evert (1976a) further prepared variable density yield tables for the jack pine cover type for three broad site classes. Evert (1976b) also developed equations and tables showing loss of volume as a result of regular mortality for the five cover types within pulpwood stands. Although Evert (1975, 1976a, 1976b) and Payandeh and Field (1986) provided needed yield information, more specific and useful yield information can be derived from such a long term and valuable permanent sample plot data set. The objective of this paper was to present variable stocking yield functions for the boreal mixedwood in Ontario based on these data.

Materials and Methods

The data set consisted of the tallies of 197 circular permanent sample plots (0.05 to 0.08 ha in size) maintained by James River/Marathon Company. Each plot had been measured

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from two to six times, mostly at 5-year intervals. Plot data included: cover type, initial stand age (average age of dominant trees of the main cover-type species), plot number, number of trees, species code, and diameter of all trees with DBH > 1.5 cm. In the last remeasurement in 1978, height and age of five to seven dominant trees per plot were also measured for estimating site index and to calculate tree volume. A detailed description of the data may be found in Payandeh and Field (1986). The estimated stand and individual tree characteristics employed are described below:

A. Basal area index

Site index (i.e. dominant stand height at an index age, e.g. 50 years for most Canadian species) is widely used to measure stand productivity in North America (Carmean 1975; Payandeh 1977; Monserud 1984a). However, by definition site index is not really applicable to uneven-aged mixedwood stands (cf. Clutter *et al.* 1983; Monserud 1984b, 1987; Payandeh 1991). To overcome such a difficulty, Payandeh and Wang (1995) developed a basal area index (BI) based on the following constrained biological growth model (Richards 1959; Lundgren and Dolid 1970; Ek 1971; Newnham 1988; Payandeh and Wang 1993) to express total basal area growth patterns over age for major timber species in Ontario:

$$BAT = b_1 BI^{b_2} (1 - e^{-b_3 t})^p$$

$$\text{with } p = \frac{BI}{b_1 BI^{b_2} \ln(1 - e^{-b_3 t_1})} \quad (1)$$

where

BAT is stand total basal area $m^2 ha^{-1}$ at age t ,
 BI is basal area index or total stand basal area ha^{-1} at a given base age, e.g., 50 years,
 t is stand age years,
 t_1 is index age,
 b_i 's are model parameters.
 p is the conditional constraint (Newnham 1988; Payandeh and Wang 1993) to satisfy the basal area index definition where:
 $BAT = BI$ at index age or 50 years
 e is base of natural logarithm.

The above polymorphic model may be used either to estimate BAT from BI and t , or to estimate BI from a given BAT and t via numerical methods (Wang and Payandeh 1993). The above model was fitted to previously published tabular data

Table 1. Estimated parameters of model [1] for major Ontario timber species

Species	Estimated parameters			RMSE	R_2
	b_1	b_2	b_3		
Black spruce	10.1416	0.4256	0.0433	0.32	0.99
Jack pine	2.8210	0.6947	0.0627	0.39	0.99
Aspen	1.9608	0.8650	0.0420	0.41	0.99
White birch	1.2194	0.9796	0.0679	0.46	0.99
White spruce	2.8339	0.7686	0.1044	1.92	0.99
Tolerant hardwood	5.5366	0.6626	0.0142	0.38	0.99
Red pine	8.6018	0.4461	0.0548	0.86	0.99
White pine	4.8313	0.5963	0.0502	0.53	0.99

(i.e., Plonski's normal yield tables) for the major species in Ontario (Payandeh and Wang 1995). The coefficients from this fitting were summarized (Table 1). BI should be better correlated with the main stand attributes such as stocking and volume than site index, particularly for the boreal mixedwood. As stated earlier (Payandeh and Wang 1995), this is mainly because BI is based on all trees in the stand rather than only a few site trees. In addition, unlike site index, basal area indices may be determined easily and inexpensively via horizontal point sampling or prism cruising.

B. Tree height and volume

Tree height data collected in the last remeasurement (1978) were used to develop cover type specific height-diameter equations according to the following model:

$$H = 1.37 + b_1 (1 - e^{-b_2 DBH})^{b_3} \quad (2)$$

where

H is tree height,
 DBH is tree diameter at breast height, and
 b_i 's are parameters to be estimated.

Tree heights were estimated by substituting individual tree diameters in the above model. Total (tree > 1.4 cm DBH) and merchantable volume (trees > 10 cm DBH) ha^{-1} were calculated by species using the tree volume equations of Honer *et al.* (1983), based on DBH and estimated tree height for individual trees.

Analysis and Results

The following nonlinear regression models (cf. Clutter *et al.* 1983; Monserud 1984a; Payandeh 1988) were used to express various stand yield attributes as a function of basal area index and stand age:

$$H = b_1 BI^{b_2} (1 - e^{-b_3 t})^{b_4 BI^{b_5}} \quad (3)$$

$$N = b_1 BI^{b_2} t^{b_3} \quad (4)$$

$$BAT = \frac{b_1 BI^{b_2}}{1 + \exp(b_3 + b_4 \ln(t) + b_5 \ln(BI))} \quad (5)$$

$$BAM = \frac{b_1 BA_t^{b_2}}{1 + \exp(b_3 + b_4 \ln(t) + b_5 \ln(BI))} \quad (6)$$

$$VT = \frac{b_1 BI^{b_2}}{1 + \exp(b_3 + b_4 \ln(t) + b_5 \ln(BI))} \quad (7)$$

$$VM = \frac{b_1 V_t^{b_2}}{1 + \exp(b_3 + b_4 \ln(t) + b_5 \ln(BI))} \quad (8)$$

Where

H is stand height (m),
 N is stand density (trees ha^{-1}),
 BAT is total basal area ($m^2 ha^{-1}$),
 BAM is merchantable basal area ($m^2 ha^{-1}$),
 VT is total volume ($m^3 ha^{-1}$),
 VM^{-1} is merchantable volume ($m^3 ha^{-1}$).

Table 2. Estimated parameters for various yield attributes for the boreal mixedwood within the James/River Marathon Company limits

Yield characteristic	b_1	b_2	b_3	b_4	b_5	R_2	RMSE
Stand height	28.2786	-0.0221	-0.0176	1.9514	-0.0741	0.48	3.50
Stems ha^{-1}	364661.8924	0.0101	-1.1852			0.52	1569
Total basal area	12.9024	-1.9774	-1.8540			0.83	3.90
Merch. basal area	0.9711	1.0014	16.4496	3.1089	-1.7072	0.92	3.36
Total volume	15.0185	1.0114	4.5222	-1.2009	0.1416	0.74	60.16
Merch. volume	1.6028	1.1866	0.8770	-0.0172	0.1617	0.98	17.03

Table 3. Various yield attributes for the boreal mixedwood of northcentral Ontario within the James River/Marathon Company limits (basal area index = 20 $m^2 ha^{-1}$)

Stand age (yr)	No. of trees ha^{-1} (>1.5 cm)	Avg. ¹ DBH (>1.5 cm)	Avg. ht (m)	Basal area ($m^2 ha^{-1}$)		Volume ($m^3 ha^{-1}$)		PAI ($m^3 ha^{-1}$)	MAI ($m^3 ha^{-1}$)
				(>1.5 cm)	(>10 cm)	(>1.5 cm)	(>10 cm)		
20	10791	2.8	4.0	6.5	0.7	64.0	47.4	0.00	2.37
30	6673	4.7	6.6	11.7	3.6	92.3	73.5	2.61	2.45
40	4745	6.6	9.1	16.3	8.5	116.1	97.0	2.35	2.43
50	3643	8.4	11.5	20.0	13.6	136.2	117.5	2.05	2.35
60	2935	9.9	13.6	22.8	17.8	153.1	135.3	1.78	2.26
70	2445	11.4	15.4	24.9	21.0	167.5	150.8	1.55	2.15
80	2087	12.7	17.1	26.5	23.5	179.8	164.4	1.36	2.01
90	1815	13.9	18.5	27.7	25.2	190.3	176.2	1.18	1.96
100	1602	15.1	19.7	28.6	26.6	199.5	186.6	1.04	1.87
110	1431	16.2	20.7	29.4	27.6	207.6	195.8	0.92	1.78
120	1291	17.2	21.6	30.0	28.4	214.6	204.0	0.82	1.70

¹Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees ha^{-1} to maintain full compatibility.

Table 4. Various yield attributes for the boreal mixedwood of northcentral Ontario within the James River/Marathon Company limits (basal area index = 30 $m^2 ha^{-1}$)

Stand age (yr)	No. of trees ha^{-1} (>1.5 cm)	Avg. ² DBH (>1.5 cm)	Avg. ht (m)	Basal area ($m^2 ha^{-1}$)		Volume ($m^3 ha^{-1}$)		PAI ($m^3 ha^{-1}$)	MAI ($m^3 ha^{-1}$)
				(>1.5 cm)	(>10 cm)	(>1.5 cm)	(>10 cm)		
20	10835	4.0	4.2	13.4	2.7	92.2	69.3	0.00	3.47
30	6701	6.3	6.8	21.1	9.9	133.5	108.2	3.89	3.61
40	4765	8.4	9.3	26.4	17.9	168.8	143.5	3.53	3.59
50	3658	10.2	11.6	30.0	24.0	198.6	174.6	3.11	3.49
60	2947	11.8	13.7	32.4	28.1	224.0	201.9	2.73	3.37
70	2455	13.3	15.5	34.0	30.8	245.7	225.7	2.38	3.22
80	2095	14.6	17.1	35.2	32.7	264.3	246.6	2.09	3.08
90	1822	15.9	18.5	36.0	33.9	280.4	265.0	1.84	2.94
100	1609	17.0	19.7	36.6	34.9	294.5	281.2	1.62	2.81
110	1437	18.1	20.7	37.1	35.5	306.8	295.6	1.44	2.69
120	1296	19.2	21.6	37.5	36.1	317.6	308.4	1.28	2.57

²Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees ha^{-1} to maintain full compatibility.

These models were fitted to the initial measurement (only) of the 197 permanent sample plots.

The estimated parameters for the yield models were summarized for the various stand attributes along with their respective fit statistics (Table 2). With the exception of stand height and trees ha^{-1} , the equations for the other stand attributes produced good to excellent fits, as indicated by R^2 and RMSE values. Yield tables generated from the solution of these regression equations (Tables 3–5) provide various yield attributes for the boreal mixedwood type within the James River/Marathon Company limits, for basal area indices of 20, 30, and 40 $m^2 ha^{-1}$.

Application

Application of basal area indices for major Ontario timber species calculated using the coefficients (Table 1) to single species, even-aged and nearly fully stocked stands should be straight forward. In the case of uneven-aged multi-species stands

such as the boreal mixedwood, one needs to obtain an estimate of stand total basal area, preferably via point sampling (or prism count) and an approximate stand age for the main cover type (e.g. black spruce, jack pine, aspen (poplar) or tolerant hardwood). Model (1) and its estimated coefficients (from Table 1) can then be used to calculate basal area index of the stand (or the main cover-type). If several species are represented fairly equally, separate basal area index/species component should be calculated and summed up to obtain the stand BI. Having the basal area index and stand age (average age of the main cover type species or the weighted average of the main species components) estimated in this way, then one may solve the remaining equations (3–8) for the species/cover-type of interest.

The yield equations presented here possess similar accuracy to those in previous reports (e.g., Evert 1975, 1976a, 1976b; Payandeh and Field 1986; Payandeh 1988), in addition to being more suitable for the boreal mixedwood. Application of basal area index in developing the yield equations has several

Table 5. Various yield attributes for the boreal mixedwood of northcentral Ontario within the James River/Marathon Company limits (basal area index = 40 m² ha⁻¹)

Stand age (yr)	No. of trees ha ⁻¹ (>1.5 cm)	Avg. ¹ DBH (>1.5 cm)	Avg. ht (m)	Basal area (m ² ha ⁻¹)		Volume (m ³ ha ⁻¹)		PAI (m ³ ha ⁻¹)	MAI (m ³ ha ⁻¹)
				(>1.5 cm)	(>10 cm)	(>1.5 cm)	(>10 cm)		
20	10867	5.1	4.3	22.1	6.5	119.3	90.7	0.00	4.54
30	6720	7.7	6.9	31.3	18.5	173.5	142.2	5.15	4.74
40	4779	9.9	9.5	36.8	28.3	219.9	189.2	4.70	4.73
50	3668	11.8	11.8	40.0	34.4	259.5	231.0	4.18	4.62
60	2955	13.5	13.8	42.0	38.1	293.3	267.8	3.68	4.46
70	2462	15.0	15.6	43.3	40.4	322.3	300.1	3.23	4.29
80	2102	16.4	17.2	44.2	41.9	347.3	328.5	2.84	4.11
90	1828	17.7	18.5	44.9	42.9	368.9	353.6	2.51	3.93
100	1613	18.9	19.7	45.3	43.6	387.9	375.8	2.22	3.75
110	1441	20.1	20.7	45.7	44.1	404.6	395.5	1.97	3.60
120	1300	21.2	21.5	46.0	44.5	419.3	413.2	1.77	3.44

¹Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees ha⁻¹ to maintain full compatibility.

advantages: 1) it serves as a valid measure of utilized site productivity equally or better correlated with stand attributes compared to site index (Payandeh and Wang 1995) mainly because BI is based on all trees within the stand rather than only a few site trees; 2) it produces variable stocking yield tables suitable for uneven-aged mixed species cover-types; and 3) unlike site index, basal area index may be estimated quickly and inexpensively via horizontal point sampling or prism cruising. Additional comparison of basal area indices and site indices may be required prior to developing basal area index and variable stocking yield functions for other species and regions. If such comparisons show a definite superiority of basal area index over that of site index, development of a new family of growth and yield models and tables based on basal area index will be in order. In addition to the boreal mixedwood, such yield tables should prove valuable for disturbed hardwood stands in southern Ontario and particularly to those in the eastern United States.

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