The FORESTRY CHRONICLE

Vol. XIII

JUNE, 1937

No. 2

REFINEMENTS IN PLOTTING AND HARMONIZING FREEHAND CURVES.

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THE plotting of free-hand curves, which is so frequent a practice with foresters, has of late years fallen very much into disrepute. The growth of interest in statistical methods has led to a feeling that unless a curve be calculated algebraically by the method of least squares, little reliance is to be placed on it. The labor of least squares calculations for anything but linear regression makes it worth while to consider means for securing adequate precision by the freehand method.

FITTING LINEAR REGRESSION LINES

For linear regression lines, the least squares method is relatively easy and is definitely to be preferred where any great degree of precision is desired. A fairly satisfactory line may be secured however by dividing the data into three groups, compromising between having the same number of observations and having the same range of the independent variable in each group. More than three groups are of no advantage. Three points may then be plotted corresponding to the means of the three groups, and an approximately correct linear regression line may then be secured by drawing a straight line through the point representing the general mean of all the data and parallel to the line joining the points representing the means of the two outer groups. If the point corresponding to the mean of the middle group is very far to the right or left of the point corresponding to the general mean, the line may need to be tilted slightly to pass nearer to it, but its effect on the slope of the line will be slight on account of its being so near the center of the distribution. In every case the line must pass through the point representing the general mean since only such a line will satisfy the condition that the sum of the values read from the curve will equal the sum of the observed values, which is the general condition that any linear regression line must satisfy. This proposition may be proved algebraically.

This method of drawing a linear regression line is illustrated in Fig. 1. The basic data are the same as for Figs. 2 and 3. As in Fig. 3, a line fitted by the method of least squares is shown dotted. The crosses indicate the means of three groups and the double circle, the general mean.

The fit of a linear regression line can in practice be improved materially only by the standard least squares calculation. The method of balancing next to be described for curved regression lines is of little assistance here.

BALANCING CURVED REGRESSION LINES

For single curved regression lines, results that are reasonably satisfactory and that in most cases are as precise as the data warrant, may be secured by a process of balancing. An outline of the method of balancing a curve is given by Chapman and Demeritt.* What follows is merely a refinement of the same idea. The illustrative data used are height-diameter measurements of White Pine at Toronto.

The data should first be sorted into unit classes of the independent variable. These classes should all include equal ranges of the independent variable. Average values of both variables should be computed for each class and points plotted. The computation of these averages and all the succeeding work are illustrated in Table 1 which has been found to be a very satisfactory form to follow with all types of data.

It is obviously impossible to draw through these points a curve whose position would be even approximately correct without some additional technique. It has been the attempt to do so that has thrown the freehand curve into disrepute. A first approximation may however be drawn fairly precisely by grouping the individual classes into three to six larger groups and plotting the averages of these groups. In selecting the groups, the graph should be carefully examined and groups selected whose averages will be approximately on a smooth curve by including in each group individual classes with both

Fig. 1—Fitting a linear regression line.

(Numbers adjoining points indicate number of sample trees)

Fig. 2—Fitting a curved regression line.

*Chapman & Demeritt—Elements of Forest Mensuration, pp. 79-80.

TABLE I
Form of Computations for Balancing a Curve

D Class	n	ΣD	$\bar{D}$	ΣH	$\bar{H}$	H'e	nH'e	Σd	$\bar{d}$	Cor- rec- tion	Total Cor'n	Hc	nHc
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
9	6	55.5	9.2	386	64.3	58.5	351.0			+1.0	+6.0	59.5	357.0
10	4	40.3	10.1	241	60.2	62.0	248.0			+0.7	+2.8	62.7	250.8
11	7	76.5	10.9	432	61.7	65.2	456.4			+0.5	+3.5	65.7	459.9
12	7	84.6	12.1	484	69.1	67.6	473.2			+0.3	+2.1	67.9	475.3
13	8	102.7	12.8	586	73.7	69.6	556.8			+0.2	+1.6	69.8	558.4
14	9	126.3	14.0	611	67.9	71.0	639.0			+0.2	+1.8	71.2	640.8
15	5	75.4	15.1	363	72.6	71.9	359.5			+0.2	+1.0	72.1	360.5
16	7	112.2	16.0	516	73.7	72.5	507.5			+0.2	+0.6	72.7	508.9
17	6	101.6	16.9	450	75.0	72.9	437.4			+0.1	+0.6	73.0	438.0
18	2	36.1	18.0	140	70.0	73.2	146.4					73.2	146.4
19	4	76.3	19.1	284	71.0	73.4	293.6					73.4	293.6
20	3	60.7	20.2	222	74.0	73.5	220.5					73.5	220.5
21													
22	1	21.8	21.8	69	69.0	73.6	73.6					73.6	73.6
GROUP TOTALS													
9-12	24	256.9	10.7	1543	64.2		1528.6	+14.4	+0.6		+14.4		1543.0
13-15	22	304.4	13.8	1560	70.8		1555.3	+4.7	+0.2		+4.4		1559.7
16-22	23	408.7	17.8	1681	73.2		1679.0	+2.0	+0.1		+2.0		1681.0
All	69	970.0	14.1	4784	69.3		4762.9	+21.1			+20.8		4783.7
D—diameter					H—total height								
ΣD —sum of diameters					ΣH —sum of heights					Σd —net sum of deviations			
$\bar{D}$ —mean diameter					$\bar{H}$ —mean height					$\bar{d}$ —mean net deviation			
n—number in class					H'e—height read from preliminary curve					Hc—curved height after balancing.			

high and low averages. Three classes whose average points form a triangle, or four forming a quadrilateral figure, will usually form satisfactory groups. The center of gravity of such geometrical figures can be judged sufficiently closely by eye to make it possible in advance to be reasonably sure that the group average points will lie in the course of a smooth curve.

The averages of the individual diameter classes are shown by circles in Fig. 2 and of the groups by crosses. A curve may now be fitted to the group averages. When there are only three groups, as in the present case, this curve should pass directly through the average points unless this would give a degree of curvature that judgment indicates to be too great. A slight degree of curvature indicated by data where the individual observations show wide variation all along the curve, should be disregarded and a linear regression line drawn. If the curvature indicated by three average points appears to be rather extreme, it is better to draw a curve with somewhat less curvature rather than to draw a curve directly through the points. An algebraic calculation would usually in such circumstances also give a curve with extreme curvature. The freehand method simply affords an opportunity to exercise reasonable judgment based on a knowledge of the amount of curvature ordinarily exhibited by data of the kind that is being handled. The individual class-average points will indicate to some extent the definiteness with which the curvature is indicated, and if the number of observations is small, the individual observations should be plotted to show how definitely the trend is defined.

Readings should next be taken from this preliminary curve on ordinates corresponding to the exact average diameters of the diameter classes (col. 7). These readings should then be multiplied by the number of observations in each class (col. 8) and the products totalled by groups. The group totals should then be subtracted from the total of the observed values (col. 5), and the differences entered in col. 9.

The reason for taking readings from the curve only at the average diameters of the trees in the different diameter classes instead of at the exact diameters of the individual trees is that that corresponds to the way in which a height curve is used. In practice only the average heights of the trees in the different diameter classes are used. When therefore readings are taken only at points on the curve corresponding to the average diameters of the diameter classes, the products of these readings times the numbers of trees in the classes represent the sums of the curve values. The differences between the sums of the observed values and the sums of the curve values are then equal to the net totals of the deviations of the individual values from the curve heights for their respective diameter classes. For the curve as a whole

and also for each section of it, adjustments should be made so that the sum of the curve values will equal the sum of the observed values. Since the sums of the curve values for each group were subtracted from the sums of the observed values, a plus difference will indicate that the curve should be raised or the curve values increased, and a minus difference that the curve should be lowered or the curve values decreased.

A special point should be noted here. If the preliminary curve is drawn directly through the average point of any group it will inevitably be too low in the case of a curve convex from above as in the example. It was pointed out in discussing linear regression that if a straight line be drawn through the point representing the general average of any data, the sum of the values read off such a line will equal the sum of the observed values. This will be true no matter what the slope of the line. Let us imagine such a straight line to be drawn through the point representing the average of any group so that it is tangent to the preliminary curve. In the case of a curve convex from above, the curve will of necessity be everywhere below the straight line except at the average point itself. Consequently, since the sum of the values read off the straight line over the range of the group will equal the sum of the observed values in the group, the sum of the values read off the curve cannot fail to be less. In other words, where a regression line is curved and convex from above, it should pass slightly above the plotted average points of groups of larger than unit size. The opposite effect is found where the curve is concave from above as in the case of a volume-diameter curve. In this case, a curve drawn through the average point of a large group will be too high. It will be seen that in the illustrative example, the curve is too low in all three groups as might be expected.

Correction for this tendency is best made when balancing the curve since the error is too small to allow for accurately when drawing the preliminary curve. The corrections made in balancing the curve are shown in col. 11, and the total effect of the corrections in col. 12. The net sum of the corrections as given in the bottom line of col. 12 should as nearly as possible equal the net sum of the deviations as given on the same line in col. 9. When the preliminary curve is drawn directly through the average points of the groups, as in the example, this equality should also obtain for the individual groups. When there are more than three groups, it will often not be possible to draw a smooth curve through the average points of all the groups. In that case, there need not be an exact correspondence between the total corrections and the net totals of the deviations of the individual groups, but they should balance for each section of the curve. When there are only three average points plotted, and the preliminary curve is made straighter than would be a

curve drawn directly through the points, the curve, if convex, should be drawn above the two end points and below the centre point in a manner somewhat similar to the manner of fitting a straight line to three average points. When that is done, the end groups will have negative net totals of deviations and the middle group a positive net total. In making corrections no attempt should be made to alter this general situation. The grand total of the corrections should however be made to balance the grand net total of the deviations. However if there is a difference between the net totals of the deviations of the two end groups, the difference should be adjusted by making a correspondingly larger total of corrections to the group with the larger net total of deviations. The effect of this would be equivalent to having the curve pass approximately the same distance away from each of the end points.

In order that the final curve may be a smooth curve, as was the preliminary curve, the corrections should form a gradual series, and if no correction is to be made to part of the curve, the corrections should gradually taper off to zero, as is done in the example. This will ensure that the balancing operation does not interfere with the smoothness of the curve.

The corrections made will usually result in values which if plotted will lie too close to the preliminary curve to justify actually drawing a second curve. In col. 13 are given values secured by adding the corrections (col. 11) to the values of the preliminary curve (col. 7). Column 14 gives the proof that the sum of these values, multiplied by the number of observations in each class, actually equals the sum of the original values. In practice these two columns are unnecessary.

The values in col. 13 are curved values corresponding to the exact average diameters of the trees in each diameter class. What are usually required are the curved values corresponding to integral values of the independent variable

Fig. 3—Comparison of curves fitted graphically and by least squares.

—in this case to diameters of exactly 9.0, 10.0 inches, etc. Values which allow for the corrections made in balancing the curve may be secured by reading off values from the preliminary curve corresponding to integral values of the independent variable and applying to them the corrections of col. 11. This can be done more accurately in this way than by actually drawing the final curve.

In Fig. 3, the final curved values from Table 1 are plotted as a solid

line and a second degree parabola fitted to the data as a dotted line. The standard errors of the curves were respectively 6.79 and 6.72 feet, a difference of less than one-tenth of a foot. This difference is wholly accounted for by differences in deviations in the first diameter class. The parabola has the disadvantage of curving downwards at the right hand end, which trend is clearly indicated by the data but not sufficiently to justify drawing the curve in that way. The parabola also flattens the bend in the middle of the curve, which may or may not be justified. There is a pronounced tendency for height-diameter curves of intolerant species to be steep over the range of the smaller diameters and flat over the large diameters with a fairly sharp bend in the centre at approximately the dividing line between the intermediate and co-dominant trees. It is a disadvantage of curves calculated by the method of least squares that they tend to even off pronounced bends of this sort, being restricted in their shape by the mathematical formula selected. On the other hand, they show extreme trends near the ends. While a freehand curve will probably seldom show a smaller standard error than one calculated by least squares, the degree of precision of the latter method may be approached very closely and greater latitude is allowed in choosing a suitable shape of curve.

A special problem arises in the use of a height curve for an individual permanent sample plot which has been balanced in the manner outlined above. The diameters of the trees on such a sample plot are usually measured to tenths of inches, and where the number of trees in an individual diameter class is limited, their average diameter may not correspond to the mid-point of the diameter class. For strict accuracy in such a case, the exact average diameter of the trees in the diameter class should be computed and the corresponding height read off the curve. This is not only more convenient but theoretically more correct than reading off heights corresponding to the exact diameters of the individual trees. The practical difference is not great but theoretically if heights are to be read off the curve corresponding to individual diameters measured to tenths of inches, the curve should be balanced in a corresponding manner by reading off heights corresponding to the exact diameters of the trees whose heights form the basis of the curve. If it is desired merely to group the trees on the sample plot in diameter classes ranging for example from 7.6 to 8.5 inches, etc., and to read heights off the curve corresponding to the mid-points of those classes, the mid-points should be taken at 8.05 inches, etc., rather than at 8.0 inches.

HARMONIZING CURVED REGRESSION LINES

The method outlined above may be applied in multiple curvilinear correlation together with three additional steps not ordinarily carried out in such work, and by these means, the precision of the work may be materially in-

creased. An ordinary volume table prepared by the conventional method or method of harmonized curves is the example most familiar to foresters, but the technique may be employed with any data where the method of harmonized curves is used.

In this method of constructing volume tables, the data are first divided into height classes and volume curves plotted over diameter for each height class. Then volumes read off at integral diameters are plotted over height in order to even off the spacing of the first set of curves and to permit reading final volumes corresponding not only to integral diameters but also to integral heights.

There is however a definite tendency for the height of the trees in a given height class to be greater for the large diameters than for the small. Chapman and Demeritt propose that in the plotting of volume over height, the average height of the sample trees in the size-class in question be used as a basis for the plotting. The volumes used in this plotting are however read off curves in which much of the irregular variation in volume has been smoothed out. The deviations in volume from the first set of curves are to a large extent due to irregularities in the average heights of the trees in the individual size-classes. It would be better therefore if the irregularities in height be themselves smoothed off by plotting for the individual height classes a set of curves of height over diameter, as in Fig. 4. Then in plotting volume over height as in Fig. 5, the volumes read off the first set of curves for each integral diameter can be plotted over heights read off the curves in Fig. 4, also for integral diameters. It may be noted in Fig. 5 that the points representing the average volumes of the trees in successive diameter classes but in the same height class, follow definite trends similar to the trends of the individual curves in Fig. 4.

The deviations of the heights plotted in Fig. 4 from the smooth curves will be found in the majority of cases to be similar to the deviations in volume from the first set of volume curves; i.e. where the average volume of a size-class is higher than the general trend, the average height of the trees in that size-class will generally be found also to be high. In drawing the height and volume curves over diameter, the two operations should be compared and similar treatment given in the two cases. Thus if the volume curve be drawn below the point for a certain size-class, the height curves should similarly be drawn below it if there appears to be any indication that the point lies above the general trend.

In plotting both these two first sets of curves, each curve should be individually balanced according to the methods outlined above. Since a considerable amount of freedom is allowed in regard to the degree of curvature that

Fig. 4—Height on diameter by height classes.

Fig. 5—Volume on height by diameter classes.

may be allowed in a curve advantage may be taken of this in harmonizing the shape of the curves, but each one should be independently balanced. If this is done, there should not be much difficulty in keeping the aggregate net deviation of the volume table within reasonable limits. Very little effort should however be made to make the height curves the same shape.

The second and third additional steps are in connection with the drawing of the curves of volume over height by diameter classes. Two difficulties will be encountered at this stage—first in making the interval between the successive curves increase in a regular manner, and second in making the steepness of the curves increase regularly. The first difficulty may be almost entirely overcome by plotting curves of both volume and height over diameter alone and reading off them the general average volume and the general average height for each integral diameter. If then these curved volumes are plotted in Fig. 5 over the curved heights for the respective diameter classes points will be secured through which the individual curves of Fig. 5 should pass, and this will ensure their being evenly spaced. These points are shown as circles in Fig. 5.

To explain the theoretical basis of this step, let it be assumed first that only one reading be made from each of the curves in Fig. 5, namely a reading corresponding to the curved height of each diameter class and that the curves are linear as in Fig. 5. These readings will of course be made at the points represented by the circles in Fig. 5 and will be the curved volumes for the different diameter classes taken originally from the general curve of volume over diameter. If each of these readings were multiplied by the number of trees in its diameter class, the sum of all these products would equal the sum of all the actual tree volumes. This would be obvious had readings off the general curve of volume over diameter been taken at the exact average diameter of each diameter class instead of at integral diameters, because in balancing the general curve it would have been ensured that the sum of the curve volumes when multiplied by the corresponding number of trees would equal the sum of the actual volumes. However taking readings at integral diameters would cause only random variations which would balance, so that the equality stated above would hold.

Now if instead of taking only one single reading off the curve for a particular diameter class, readings were made corresponding to the height of each individual tree in the class, each reading would differ from the previous single reading by b times the deviation in height of the tree from the curved height for the diameter class, i.e. by $b(H-H_c)$, where b is the slope of the line. The sum of all the readings for the diameter class would be.

$$b \sum (H-H_c) = b (\sum H - nH_c).$$

If this were done for all the diameter classes, the differences for the various diameter classes would correspond to the differences between the total heights of the trees in the diameter classes and n times the curved heights for the diameter classes. But in balancing the height curve, these differences would be made to balance not only over the whole length of the curve, but also for each section of it. Therefore even though the b 's, i.e. the slopes of the curves would vary for the different diameter classes, if this variation followed a regular trend as it should, the differences between the sum of the volumes read off the curves corresponding to the heights of the individual trees and the sum of the single readings from the curves in Fig. 5 when multiplied by the corresponding numbers of trees would also balance. In other words the sum of such individual readings would for all the diameter classes together equal the sum of the values of the single readings multiplied by the corresponding number of trees.

In the third preceding paragraph it was shown that the sum of the single readings when multiplied by the corresponding numbers of trees was equal to the sum of all the actual volumes, consequently the sum of the individual readings would likewise equal the sum of the actual tree volumes. This would be true no matter what slopes were given to the curves of volume over height, provided only that the slopes varied in a gradual regular manner from diameter class to diameter class.

It remains therefore to fix the slopes of the lines. Their slopes will be indicated by the positions of the points for the different size-classes. If no marked curvilinearity is indicated by the plotted points, in which case straight lines may be drawn, a consistent gradual increase in slope may be secured by drawing the lines back toward the origin as far as possible. Both scales of the graph should for this purpose start at zero. The lines should be drawn so that their intersections with the axes near the origin vary in a consistent and gradual way, or there may be evident a tendency for the lines to converge at a single point on or near the horizontal axis. In Fig. 5, the six inch line intersects the vertical axis at 1 cu. ft.; the eight to fourteen inch lines at points regularly spaced from 0 to 15 feet on the horizontal axis, and all the remaining lines intersect the horizontal axis at the fifteen foot mark.

CHECKING AND ADJUSTING THE FINAL TABLE

In reading the values for the final volume table from Fig. 5, it is a good plan to tabulate the values leaving spaces between successive lines and columns, and to insert in the spaces the differences between the values for successive diameters in a height class and for successive heights in each diameter class. The former differences should keep getting greater at an increasing rate, and the latter should if straight lines have been drawn in Fig. 5, tend to be con-

stant. Small changes in the values read from the graph may be made to achieve these results, and this will eliminate apparent inconsistencies in the table which might otherwise show up if interpolations were to be made in it. This will usually give a better final result than plotting the values back over diameter by height classes, which is the procedure usually recommended for the same purpose and there will be less tendency to reduce the agreement of the tabular values with the basic data.

If the procedure outlined is followed particularly in regard to balancing the individual curves in the first stages of the work, the aggregate net deviation should not be large. It may however be brought practically to zero by interpolating in the volume table for the average diameter and height of the trees in each size-class, multiplying by the numbers in the size-classes, and subtracting these products from the sum of the actual volumes of the trees in the size-classes which will give the net sum of the deviations of the trees in the various size-classes. Totalling these figures for different diameter groups in each height class will reveal where the tabular values are high or low, and suitable corrections may be applied to the table so as to bring the net sum of the deviations to zero and also to improve the correspondence of the different parts of the table with the basic data.

Table II illustrates how this may be done.

TABLE II								
Net Deviation of Tree Volumes from Tabular Volumes								
Diameter Group	Height Class							Total
	30	40	50	60	70	80	90	
Net Deviation								
5- 7"	+0.1	+5.0	-4.4					+0.7
8-10"		+1.7	+2.0	+10.5	+1.3			+15.5
11-13"				-1.3	+4.5	+0.6		+3.8
14-18"					+0.6	+4.0	-8.5	-3.9
Total	+0.1	+6.7	-2.4	+9.2	+6.4	+4.6	-8.5	+16.1
After Correction								
8-10"		+1.1	-2.4	+1.2	+0.1			0.0
Total	+0.1	+6.1	-6.8	-0.1	+5.2	+4.6	-8.5	+0.6

The aggregate net deviation is 16.1 cu. ft. which is 0.26 per cent. of 6091.1 cu. ft.—the total volume of all the sample trees. Adding 0.1 cu. ft. to tabular volumes of the eight and ten inch diameter classes and 0.2 cu. ft. to the nine inch class will eliminate the plus net deviation in this group and reduce the aggregate deviation to 0.6 cu. ft. This change amounts to only 1.1 per cent. of the volume of the trees affected.

Such a tabulation will indicate fairly definitely whether the steepness of the lines in Fig. 5 is correct. A positive value on the left of any horizontal

line of figures and a negative value on the right will indicate that the lines for the diameter classes in the group in question are too steep, and the reverse, that they are too flat. The indications in the above tabulation are:

5-7"—too steep,

8-10"—about right,

11-13"—too flat,

14-18"—too steep.

The indications are not consistent and no obvious changes are suggested by them. In general, positive and negative deviations are pretty well balanced over the whole table.

The net deviation of all the trees in the individual height classes is given in Table II but these figures are not very sensitive in indicating whether the regression of volume over height is correctly allowed for. Positive deviations for trees of small diameter may balance negative deviations of larger trees in the same height class—particularly in the central height classes.

SUMMARY

The technique of plotting straight and curved regression lines has been outlined, and a definite procedure for balancing the latter has been described.

The application of the balancing of curved regression lines to the conventional method of harmonizing volume curves has been shown, and three additional steps suggested for improving the precision of that operation.

Finally, the method of computing and tabulating the net deviation of the basic data from the final tabular volumes is given, and suggestions made as to how any aggregate net deviation may be eliminated and any minor errors in trends corrected.

The procedures outlined are of general application to multiple curvilinear correlation problems involving three variables provided the number of observations is sufficiently large and the trends well enough defined to permit the plotting with a sufficient degree of precision of the separate curves required in the first stages of the harmonized curves method.

PERFECTIONNEMENT DES METHODES EMPLOYEES POUR TRACER ET HARMONISER LES COURBES DE CERTAINS CALCULS GRAPHIQUES

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L'auteur a déjà indiqué la technique à suivre pour mettre en plan et tracer des courbes de certains calculs graphiques, de même que la méthode pour harmoniser les courbes obtenues de cette façon. Lorsqu'il s'agit d'une ligne droite il suggère que la ligne soit subordonnée non seulement à trois points moyens mais aussi de la faire passer par un quatrième point qui

représenterait la moyenne de tous les résultats sur lesquels ces calculs sont basés. Lorsqu'il s'agit d'une ligne courbe, les moyennes de trois à six points moyens reuvent être mises en plan ce qui permettra d'esquisser une courbe préliminaire. Cette courbe sera ensuite régularisée en suivant la méthode indiquée dans la table No. 1; de cette façon, on sera certain que la somme de toutes les valeurs trouvées au moyen de cette ligne courbe seront égales au total des valeurs observées et que cette égalité sera aussi constatée pour chaque section de la dite courbe. On obtiendra ainsi des courbes ayant une précision suffisante sans qu'il soit nécessaire pour obtenir les mêmes résultats d'employer les procédés laborieux qu'exigent la méthode dite les plus petits carrés.

Plusieurs perfectionnements dans l'harmonisation des courbes sont énumérés qui se rapportent à la méthode habituelle de construire des tarifs de cubage. En premier lieu, les courbes individuelles doivent être régularisées indépendamment avant d'harmoniser l'ensemble. La plus importante des opérations additionnelles suggérées est la mise en plan d'un groupe de courbes de hauteurs basées sur les diamètres des arbres dans chaque catégorie de hauteurs, suivant qu'indiqué au dessin No. 4. Grâce à cette méthode, la tendance pour la hauteur des arbres comprises dans une catégorie de hauteur d'être plus élevées quand le diamètre augmenté peut être correctement allouée lorsque les courbes sont mises en plan (voir dessin No. 5).

L'auteur indique également d'autres acheminements additionnels qui seront utiles pour mettre en plan les courbes du dessin No. 5 et pour ajuster les valeurs servant à construire le tarif de cubage final.