

A simple bole model¹

D. W. ORMEROD

Mblambanyati
Swaziland

Most equations developed for volume and upper-bole diameter estimates are of an empiric, rather than geometric, origin. As such, these equations are of limited use. For instance, in volume table construction for different utilization standards, separate computations, and possibly different models, are required for each standard. A general stem profile model, that can be integrated to give a volume equation, is desirable. The deficiencies of the empiric approach, and the rationale for using a profile model, are well developed in the introductory comments of Bruce *et al.* (1968); being obvious, they need not be elaborated here. The author considers that the simple and flexible geometric model described in this paper may serve as a suitable basis for developing volume equations for inventory, and for use in research.

Upper-hole diameters, of tree species of typically excurrent form, may be estimated by the following equation;

$$d = D \left[\frac{H - h}{H - k} \right]^p, p > 0 \quad (1)$$

where; d is the estimated diameter, h is the height of estimation, H is the total height, D is the measured diameter at height k , and p is the fitted exponent.

Equation 1 is so conditioned that when $h = k$, $d = D$, and when $h = H$, $d = 0$. If the fitted exponent p is less than one the shape of the hole will be parabolic, and if greater than one, then neiloidal. This equation was previously developed by the author (Ormerod 1971) for use in a purely geometric simulation of tree component interaction during

¹This study, completed in 1971 at the University of British Columbia, forms part of the continuing research on tree taper, directed by Dr. A. Kozak of UBC's Faculty of Forestry, and assisted by the National Research Council of Canada.

thinning extraction, where for reasons of computation economy it was necessary to use a very simple function. The ratio $(H - h) / (H - k)$ was used as an independent variable in the taper equation described by Bruce *et al.* (1968).

The simple model of Equation 1 may not provide an adequate description of the bole, due to changes in form along the length. A better description may result from the use of a modified form of Equation 1 as a step function. Separate exponents have to be fitted for each step. The equation follows:

$$d = (D_i - C_i) \left[\frac{H_i - h}{H_i - k} \right]^{p_i} + C_i, p_i > 0 \quad (2)$$

where; H_i is the height to top of section i , C_i is the section diameter intercept, D_i is the section measured diameter at height k_i , and p_i is the fitted exponent on the closed interval [$h = H_{i-1}$, $h = H_i$].

In Fig. 1, illustrating a two-section case, it should be noted that if $D_2 \neq C_1$ then the estimate d at $h = H_1$ given by the equation for section 1 may be significantly different from the estimate given by the equation for section 2.

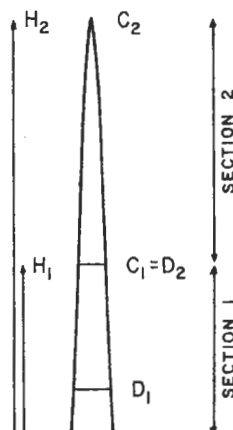


Fig. 1. A two-section case.

Considering Equation 2 and omitting the subscript *i* for the sake of clarity, the following useful derivations are made:

1/ Estimating the height *h* for a given diameter *d* by

$$h = H - \left[\frac{d - C}{D - C} \right]^{1/p} (H - k) \quad (3)$$

2/ Estimating the volume *v* between limits *a* and *b* on *h*;

Let $d = f(h) = \text{Equation 2}$

$$\begin{aligned} \text{Then } v &= \pi \int_a^b \left[\frac{f(h)}{2} \right]^2 \delta h \\ &= \frac{\pi}{4} \left[x + y + C^2 h \right]_a^b \end{aligned} \quad (4)$$

$$\text{where; } x = - \frac{(D - C)^2 (H - h)^{2p+1}}{(2p + 1) (H - k)^{2p}}$$

$$y = - \frac{2C(D - C) (H - h)^{p+1}}{(p + 1) (H - k)^p}$$

The methods of Kozak (1969) and Kozak *et al.* (1969) and the British Columbia Forest Service (BCFS) taper curve data used by them, were used to test Equation 2. In these tests it was assumed that all boles had a single taper inflection point at .3 of the total height. This assumption was based on a careful inspection of the taper curves and because

it suited the data. Separate exponents were then fitted for the sections below and above that point, by regression of the logarithmic transformation of Equation 2. These exponents were fitted over the entire range of diameters and heights for each species.

The data used provided diameters inside bark (dib) at 1 ft, 4.5 ft, and at deciles of total height, and the diameter outside bark at 4.5 ft (dbhob). These data are measurements taken directly from the smoothed taper curves of the BCFS, which were prepared from large samples of trees covering a wide range of sizes and sites.

The taper equations fitted in this study were compared by determining the standard errors of the estimates of inside-bark diameters, with the same data that were used to fit the parameters. It should be noted that for a step function, each separate exponent fitted will remove one degree of freedom.

Kozak *et al.* (1969) indicated that butt flare of large trees resulted in large standard errors of the diameter estimates in tests of their equations, and to reduce these errors they assumed that dib at 1 ft was dbhob. This assumption is acceptable because the consequently better fit will give more accurate estimates in the upper portion of the bole.

Tables 1 and 2 provide a comparison of Equation 2 with the equation of Kozak *et al.* (1969). In these tables note that:

1/ Equation A is

$$d = \text{dbhob} \left[\sum_{i=1}^2 b_i \left(1 - \frac{h}{H} \right) \right]^{\frac{1}{2}}$$

TABLE 1. Standard errors of the inside-bark diameter estimates (in inches)

Species	Number of BCFS curves	Equations		
		A Adjusted	B Un-adjusted	B Adjusted
Alder C, M ^a	92	.84	.73	.37
Aspen C, M	53	.59	.47	.24
Balsam C, M	85	.90	1.56	.55
Balsam In, M	85	.59	.82	.29
Birch In, M	55	.32	.91	.18
Cedar C, M	114	2.13	2.70	.75
Cedar C, Im	134	1.61	1.99	.63
Cedar In, M	127	1.30	2.23	.56
Cottonwood				
C + In, M	92	.84	.72	.51
Douglas-fir C, M	114	1.54	1.04	1.17
Douglas-fir In, M	160	1.33	1.18	.89
Hemlock C, M	118	.98	1.30	.54
Hemlock C, Im	128	1.16	1.26	.47
Hemlock In, M	104	.73	.89	.53
Larch In, M	65	1.33	.92	.93
Lodgepole pine				
C + In, M	148	.71	.66	.55
Maple C, M	48	.41	.37	.17
Spruce C, M	378	2.34	4.16	1.43
Spruce In, M	93	.71	1.18	.28
White pine				
C + In, M	81	1.01	1.27	.79
Yellow cedar				
C + In, M	50	.78	.77	.44
Yellow pine				
C + In, M	124	1.02	1.09	.89

^aC is coastal, In is interior, M is mature and Im is immature

TABLE 2. Equation parameters

Species	Equations				
	A Adjusted		A Unadjusted		B Adjusted ^a
	b ₁	b ₂	p ₁	p ₂	p ₁
Alder C, M	.5448	.4480	1.9383	.7494	1.1990
Aspen C, M	.5144	.4433	2.0394	.7208	1.7450
Balsam C, M	.6705	.2966	4.4022	.6243	1.5482
Balsam In, M	.4947	.4812	2.9509	.7118	1.3343
Birch In, M	.2171	.7878	1.1948	.8873	.9513
Cedar C, M	.0001	.9223	4.8229	.7384	1.6790
Cedar C, Im	.2546	.7255	1.7650	.7494	.9889
Cedar In, M	.1382	.7923	3.8026	.7187	1.5621
Cottonwood C + In, M	.3740	.5557	2.3979	.7824	1.9715
Douglas-fir C, M	.2246	.5829	3.4609	.6187	4.2022
Douglas-fir In, M	.3087	.5364	2.8780	.6655	3.1655
Hemlock C, M	.4137	.5259	3.8978	.6975	1.8617
Hemlock C, Im	.3726	.5981	1.0461	.7800	.9299
Hemlock In, M	.5539	.3904	2.8623	.6689	1.9446
Larch In, M	.3282	.4918	2.3672	.5911	2.3590
Lodgepole pine					
C + In, M	.7172	.2566	2.6284	.5661	1.3195
Maple C, M	.1720	.8410	1.7942	.8822	1.0390
Spruce C, M	.2135	.7299	4.3967	.7087	1.7662
Spruce In, M	.3964	.5682	3.4687	.6967	1.5284
White pine C + In, M	.5572	.3968	3.2043	.6404	1.6934
Yellow cedar					
C + In, M	.4317	.5589	3.4342	.6791	1.2316
Yellow pine					
C + In, M	.5440	.3310	3.6964	.6152	3.3475

^aNote that p₂ adjusted is the same as p₂ unadjusted.

which is another expression of the equation of Kozak *et al.* (1969), viz:

$$d = dbhob \left[a + b \left(\frac{h}{H} \right) + c \left(\frac{h}{H} \right)^2 \right]^{\frac{1}{2}}$$

where the coefficients are equated as follows:

$$\begin{aligned} a &= b_1 + b_2 \\ b &= -(b_1 + 2b_2) \\ c &= b_2 \end{aligned}$$

2/ Equation B is Equation 2 with $k_2 = .3$ total height.

3/ The adjustment refers to the assumption that dib at 1 ft = $dbhob$.

The accuracy of estimation of the equations described has been limited by the 'mean tree' approach. Equation 2 would be better used by fitting the exponents as functions of the more important physical and biological parameters. These may include $dbhob$, height, density, age and site. It may also be possible to group species with similar mechanical properties. This approach has not been used in this study because the objective has been to determine whether or not the model has any potential advantage for reducing the errors of upperbole diameter estimates.

The use of the step function requires the measurement, or estimation, of the diameters and heights at the section intercepts. In addition, if inside-bark diameters are to be estimated from outside-bark measurements, then the bark thickness at these measurement points must also be estimated. In trees with insignificant butt swell sufficiently accurate estimates may be obtained with a single-section model.

In conclusion, it is suggested that Equation 2, with derivations Equations 3 and 4, is a general model for excurrent tree boles. The model can provide accurate estimates, but the overall utility for inventory and research depends upon the specific accuracy requirements, and the costs allowed for measurement.

References

- BRUCE, D., R. O. CURTIS and C. VANCOEVERING. 1968. Development of a system of taper and volume tables for red alder. *Forest Sci.* 14:339-350.
- KOZAK, A., D. D. MUNRO and J. H. G. SMITH. 1969. Taper functions and their applications in forest inventory. *Forest. Chron.* 45:278-283.
- _____, 1969. More accuracy required. *Truck Log.* 25(12): 20-21.
- ORMEROD, D. W. 1971. A geometric model of skyline thinning damage. Unpub. M. F. thesis, Fac. Forest., Univ. British Columbia.



Public Service
Canada

Fonction publique
Canada

THESE COMPETITIONS ARE OPEN TO BOTH MEN AND WOMEN

FOREST PROTECTION STAFF ADVISER

Salary to \$21,586 (Under review)

Water, Lands, Forests and Environment Division
Department of Indian and Northern Affairs
OTTAWA, ONTARIO

DUTIES: — Under the direction of the Administrator of Northern Forests, develops and recommends programs, procedures and guidelines for the implementation of forest protection policies in the Yukon Territory and the Northwest Territories; provides professional, technical, operational and policy direction and assistance to the Northwest and Yukon Lands and Forest Services; develops, implements and maintains procedures for the collecting and analysis of forest fire statistics and the daily briefing of senior officials on the fire situation in the Yukon Territory and the Northwest Territories; and provides assistance as required to the Administrator of Northern Forests on overall forestry programs and projects.

QUALIFICATIONS: — Graduation with a Bachelor's Degree in Forestry with several years experience in the area of forest fire control operations. Experience in forest fire research would be desirable.

Knowledge of the English language is essential.

Please quote competition No. 73-125-06
in all correspondence.

SUPERINTENDENT, NORTHWEST LANDS AND FORESTS SERVICE

Salary to \$21,586 (Under review)
(Plus Northern Allowance)

Water, Lands, Forests and Environment Division
Department of Indian and Northern Affairs
FORT SMITH, N.W.T.

DUTIES: — Under the direction of the Regional Manager, Water, Forests and Land Division, N.W.T., plans, recommends, develops, controls and supervises renewable resource management programs dealing with forest fire control, timber management and utilization, surface land use operations, and environmental protection;

- Ensures the effective and uniform enforcement of related legislation, policies, objectives, permits and resource development agreements to ensure the proper conservation, development, and utilization of forest and land resources;
- Directs the functioning of the Northwest Lands and Forests Service in the effective implementation of the Department's forestry and land use responsibilities;
- Other related duties such as replying to public enquiries, coordinating with other federal and territorial government departments and agencies in related operations.

QUALIFICATIONS: — Graduation from a university of recognized standing with a Bachelor's degree in forestry; several years experience in the application of academic knowledge and training to directly related duties.

Knowledge of the English language is essential.

WORKING CONDITIONS

Applicable isolated posts regulations. Frequent travel throughout the Northwest Territories on chartered aircraft or other sea and land vehicles.

Please quote competition number 73-125-04
in all correspondence.

To ensure consideration, applications or a detailed résumé should be forwarded before July 6, 1973 to:

SCIENCES AND TECHNOLOGY PROGRAM,
PUBLIC SERVICE COMMISSION OF CANADA,
TOWER "A", PLACE DE VILLE,
OTTAWA, ONTARIO
K1A 0M7.