A simple bole model[1]

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Most equations developed for volume and upper-bole diameter estimates are of an empiric, rather than geometric, origin. As such, these equations are of limited use. For instance, in volume table construction for different utilization standards, separate computations, and possibly different models, are required for each standard. A general stem profile model, that can be integrated to give a volume equation, is desirable. The deficiencies of the empiric approach, and the rationale for using a profile model, are well developed in the introductory comments of Bruce *et al.* (1968); being obvious, they need not be elaborated here. The author considers that the simple and flexible geometric model described in this paper may serve as a suitable basis for developing volume equations for inventory, and for use in research.

Upper-hole diameters, of tree species of typically excurrent form, may be estimated by the following equation;

$$d = D \left[\frac{H - h}{H - k}\right]^{p}, p > 0 \qquad (1)$$

where; d is the estimated diameter, h is the height of estimation, H is the total height, D is the measured diameter at height k, and p is the fitted exponent.

Equation I is so conditioned that when $h = k$, $d = D$, and when $h = H$, $d = 0$. If the fitted exponent p is less than one the shape of the hole will be parabolic, and if greater than one, then neiloidal. This equation was previously developed by the author (Ormerod 1971) for use in a purely geometric simulation of tree component interaction during thinning extraction, where for reasons of computation economy it was necessary to use a very simple function. The ratio $(H - h) / (H - k)$ was used as an independent variable in the taper equation described by Bruce *et al.* (1968).

The simple model of Equation I may not provide an adequate description of the bole, due to changes in form along the length. A better description may result from the use of a modified form of Equation I as a step function. Separate exponents have to be fitted for each step. The equation follows:

$$d = (D_i - C_i) \left[\frac{H_i - h}{H_i - k}\right]^{p_i} + C_i, p_i > 0 \qquad (2)$$

where; H_i is the height to top of section i, C_i is the section diameter intercept, D_i is the section measured diameter at height k_i, and p_i is the fitted exponent on the closed interval $[h = H_{i-1}, h = H_i]$.

In Fig. 1, illustrating a two-section case, it should be noted that if $D_2 \neq C_1$ then the estimate d at $h = H_1$ given by the equation for section 1 may be significantly different from the estimate given by the equation for section 2.

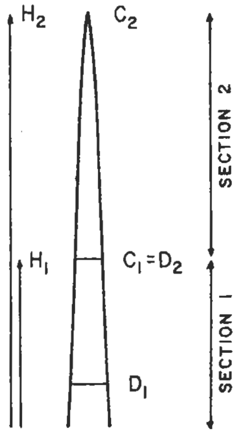

Fig. 1. A two-section case.

[1]This study, completed in 1971 at the University of British Columbia, forms part of the continuing research on tree taper, directed by Dr. A. Kozak of UBC's Faculty of Forestry, and assisted by the National Research Council of Canada.

Considering Equation **2** and omitting the subscript i for the sage of clarity, the following useful derivations are made:

1/ Estimating the height h for a given diameter d by

$$h = H - \left[\frac{d - C}{D - C}\right]^{1/p} (H - k) \qquad (3)$$

2/ Estimating the volume v between limits a and b on h;

Let d = f(h) = Equation **2**

$$\text{Then } v = \pi \int_a^b \left[\frac{f(h)}{2}\right]^2 \delta h$$

$$= \frac{\pi}{4} \left[x + y + C^2 h\right]_a^b \qquad (4)$$

$$\text{where; } x = -\frac{(D - C)^2 (H - h)^{2p+1}}{(2p + 1)(H - k)^{2p}}$$

$$y = -\frac{2C(D - C)(H - h)^{p+1}}{(p + 1)(H - k)^p}$$

The methods of Kozak (1969) and Kozak *et al.* (1969) and the British Columbia Forest Service (BCFS) taper curve data used by them, were used to test Equation **2**. In these tests it was assumed that all boles had a single taper inflection point at .3 of the total height. This assumption was based on a careful inspection of the taper curves and because it suited the data. Separate exponents were then fitted for the sections below and above that point, by regression of the logarithmic transformation of Equation **2**. These exponents were fitted over the entire range of diameters and heights for each species.

The data used provided diameters inside bark (dib) at 1 ft, 4.5 ft, and at deciles of total height, and the diameter outside bark at 4.5 ft (dbhob). These data are measurements taken directly from the smoothed taper curves of the BCFS, which were prepared from large samples of trees covering a wide range of sizes and sites.

The taper equations fitted in this study were compared by determining the standard errors of the estimates of inside-bark diameters, with the same data that were used to fit the parameters. It should be noted that for a step function, each separate exponent fitted will remove one degree of freedom.

Kozak *et al.* (1969) indicated that butt flare of large trees resulted in large standard errors of the diameter estimates in tests of their equations, and to reduce these errors they assumed that dib at 1 ft was dbhob. This assumption is acceptable because the consequently better fit will give more accurate estimates in the upper portion of the bole.

Tables 1 and 2 provide a comparison of Equation 2 with the equation of Kozak *et al.* (1969). In these tables note that:

1/ Equation A is

$$d = \text{dbhob} \left[\Sigma_{i=1}^{2} b_i \left(1 - \frac{h}{H}\right)\right]^{\frac{1}{2}}$$

TABLE 1. Standard errors of the inside-bark diameter estimates (in inches)

Species	Number of BCFS curves	Equations A Adjusted	B Unadjusted	B Adjusted
Alder C, M[a]	92	.84	.73	.37
Aspen C, M	53	.59	.47	.24
Balsam C, M	85	.90	1.56	.55
Balsam In, M	85	.59	.82	.29
Birch In, M	55	.32	.91	.18
Cedar C, M	114	2.13	2.70	.75
Cedar C, Im	134	1.61	1.99	.63
Cedar In, M	127	1.30	2.23	.56
Cottonwood C + In, M	92	.84	.72	.51
Douglas-fir C, M	114	1.54	1.04	1.17
Douglas-fir In, M	160	1.33	1.18	.89
Hemlock C, M	118	.98	1.30	.54
Hemlock C, Im	128	1.16	1.26	.47
Hemlock In, M	104	.73	.89	.53
Larch In, M	65	1.33	.92	.93
Lodgepole pine C + In, M	148	.71	.66	.55
Maple C, M	48	.41	.37	.17
Spruce C, M	378	2.34	4.16	1.43
Spruce In, M	93	.71	1.18	.28
White pine C + In, M	81	1.01	1.27	.79
Yellow cedar C + In, M	50	.78	.77	.44
Yellow pine C + In, M	124	1.02	1.09	.89

[a]C is coastal, In is interior, M is mature and Im is immature

TABLE 2. Equation parameters

Species	Equations A Adjusted b_1	A Adjusted b_2	A Unadjusted p_1	A Unadjusted p_2	B Adjusted[a] p_1
Alder C, M	.5448	.4480	1.9383	.7494	1.1990
Aspen C, M	.5144	.4433	2.0394	.7208	1.7450
Balsam C, M	.6705	.2966	4.4022	.6243	1.5482
Baslsam In, M	.4947	.4812	2.9509	.7118	1.3343
Birch In, M	.2171	.7878	1.1948	.8873	.9513
Cedar C, M	.0001	.9223	4.8229	.7384	1.6790
Cedar C, Im	.2546	.7255	1.7650	.7494	.9889
Cedar In, M	.1382	.7923	3.8026	.7187	1.5621
Cottonwood C + In, M	.3740	.5557	2.3979	.7824	1.9715
Douglas-fir C, M	.2246	.5829	3.4609	.6187	4.2022
Douglas-fir In, M	.3087	.5364	2.8780	.6655	3.1655
Hemlock C, M	.4137	.5259	3.8978	.6975	1.8617
Hemlock C, Im	.3726	.5981	1.0461	.7800	.9299
Hemlock In, M	.5539	.3904	2.8623	.6689	1.9446
Larch In, M	.3282	.4918	2.3672	.5911	2.3590
Lodgepole pine C + In, M	.7172	.2566	2.6284	.5661	1.3195
Maple C, M	.1720	.8410	1.7942	.8822	1.0390
Spruce C, M	:2135	.7299	4.3967	.7087	1.7662
Spruce In, M	.3964	.5682	3.4687	.6967	1.5284
White pine C + In, M	.5572	.3968	3.2043	.6404	1.6934
Yellow cedar C + In, M	.4317	.5589	3.4342	.6791	1.2316
Yellow pine C + In, M	.5440	.3310	3.6964	.6152	3.3475

[a]Note that p_2 adjusted is the same as p_2 unadjusted.

which is another expression of the equation of Kozak *et al.* (1969), viz:

$$d = dbhob \left[a + b\left(\frac{h}{H}\right) + c\left(\frac{h}{H}\right)^2\right]^{\frac{1}{2}}$$

where the coefficients are equated as follows:

$$a = b_1 + b_2$$
$$b = -(b_1 + 2b_2)$$
$$c = b_2$$

2/ Equation B is Equation **2** with k_2 = .3 total height.

3/ The adjustment refers to the assumption that dib at 1 ft = dbhob.

The accuracy of estimation of the equations described has been limited by the 'mean tree' approach. Equation **2** would be better used by fitting the exponents as functions of the more important physical and biological parameters. These may include dbhob, height, density, age and site. It may also be possible to group species with similar mechanical properties. This approach has not been used in this study because the objective has been to determine whether or not the model has any potential advantage for reducing the errors of upper-bole diameter estimates.

The use of the step function requires the measurement, or estimation, of the diameters and heights at the section intercepts. In addition, if inside-bark diameters are to be estimated from outside-bark measurements, then the bark thickness at these measurement points must also be estimated. In trees with insignificant butt swell sufficiently accurate estimates may be obtained with a single-section model.

In conclusion, it is suggested that Equation **2,** with derivations Equations **3** and **4**, is a general model for excurrent tree boles. The model can provide accurate estimates, but the overall utility for inventory and research depends upon the specific accuracy requirements, and the costs allowed for measurement.

References

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